Cosmology from the two-dimensional renormalization group acting as the Ricci flow

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Abstract

I describe first exploratory attempts at a top-down, first-principles calculation of macroscopic cosmology within the Standard Model (arXiv:1909.01374 [astro-ph.CO]). The 2d RG acting as the Ricci flow produces a specific 1+3d metric which describes an expanding universe that starts with a big bang then decelerates then accelerates until ending with a big blowup. This crude calculation of cosmology omits all but the gravitational field. The only energy-momentum is purely gravitational dark matter and energy. The second exploratory step adds in the SU(2) gauge field and the Higgs field. An early-time solution describes an inflating universe with cosmological CP violation during the end stage of inflation.

The RG of the 2d general nonlinear model acts as the Ricci flow

$$\Lambda \frac{\partial}{\partial \Lambda} g_{\mu\nu} = R_{\mu\nu}$$

The 2d field takes its values in a manifold M. The metric $g_{\mu\nu}$ encodes the couplings. The 2d RG drives $g_{\mu\nu}$ to a fixed point

$$R_{\mu\nu} = \nabla_\mu v_\nu + \nabla_\mu v_\mu$$

where $v^\mu(x)\partial_\mu$ is a vector field on M, an infinitesimal reparametrization $\dot{x}^\mu = v^\mu(x)$, which is a field redefinition in the nonlinear model. The rhs is the corresponding redundant perturbation (gauge transformation)

$$\mathcal{L}_v g_{\mu\nu} = \nabla_\mu v_\nu + \nabla_\mu v_\mu$$

Thus the fixed point equation expresses physical 2d scale invariance.

I have been working for a long time on a fundamental theory of physics based on a quantum version of the 2d RG (with more fields than $g_{\mu\nu}(x)$ as couplings of the 2d nonlinear model).

Among the features that appeal to me: it is a mechanism that *produces* space-time physics and it does so from the top down, from the largest length scales down to the smaller.

I have an idea of a specific 2d RG trajectory that might work, but figuring out how to calculate real observables from such an abstract framework is an intimidating project. I'd like to an idea what the target of such a calculation might look like.

So I've recently been exploring what large-distance physics — what cosmology — the 2d RG acting as the Ricci flow could produce.

I start very naively, making opportunistic assumptions:

1. at first include only the metric $g_{\mu\nu}$, no Standard Model fields
2. assume classical field theory, i.e., only the 2d RG not the quantum version of the 2d RG
3. assume space is S^3 with $\mathbf{SO}(4)$ symmetry
4. assume analyticity in time (so that the solution can be analytically continued from imaginary time where the 2d nonlinear model makes sense)

Space-time is $I \times S^3$ where I is an interval of time.

The most general $\mathbf{SO}(4)$-invariant metric is

$$g_{\mu\nu} dx^\mu dx^\nu = -F_1(t)^2 dt^2 + F_2(t)^2 ds^2_{S^3}$$

$$ds^2_{S^3} = \text{ metric of unit 3-sphere}$$

Reparametrize $F_1 dt = F_2 dT$ to go to the conformally flat form

$$g_{\mu\nu} dx^\mu dx^\nu = a(T)^2 \left(-dT^2 + ds^2_{S^3}\right)$$

The general $\mathbf{SO}(4)$ invariant vector field is

$$v^\mu(x)\partial_\mu = v^T(T)\partial_T$$

So there are two dependent variables. The useful variables are

$$f_T = a^{-1}\partial_T a \qquad v_T = g_{TT} v^T = -a^2 v^T$$

Calculate

$$R_{\mu\nu}dx^\mu dx^\nu = (-3\partial_T f_T)\, dT^2 + \left(\partial_T f_T + 2f_T^2 + 2\right) ds_{S^3}^2$$

$$(\nabla_\mu v_\nu + \nabla_\nu v_\mu)dx^\mu dx^\nu = (2\partial_T v_T - 2f_T v_T)\, dT^2 + (-2f_T v_T)\, ds_{S^3}^2$$

so the fixed point equation $R_{\mu\nu} = \nabla_\mu v_\nu + \nabla_\nu v_\mu$ is

$$-3\partial_T f_T = 2\partial_T v_T - 2f_T v_T$$

$$\partial_T f_T + 2f_T^2 + 2 = -2f_T v_T$$

which is the ode

$$\partial_T v_T = 3f_T^2 + 3 + 4f_T v_T$$

$$\partial_T f_T = -2f_T^2 - 2 - 2f_T v_T$$

There is an essentially unique solution that analytically continues to imaginary time (derivation later)

$$f_T = \frac{\cos 2T + \sqrt{3}}{\sin 2T} \qquad v_T = \frac{-\sqrt{3}}{\sin 2T} \qquad T \in (0, \pi/2)$$

$$a(T) = t_0' \sin^{1+\nu} T \cos^{-\nu} T \qquad \nu = \frac{\sqrt{3}-1}{2} = 0.3660\ldots$$

The only free parameter is the overall time scale t_0'.

in co-moving time t $\qquad ds^2 = -dt^2 + a^2 ds^2_{S^3} \qquad dt = a(T)dT$

$$\frac{t}{t_0'} = \int_0^T \sin^{1+\nu} T' \cos^{-\nu} T' \, dT' = \frac{1}{2} B_{\sin^2 T}\left(\frac{2+\nu}{2}, \frac{1-\nu}{2}\right)$$

$$t \in (0, t_{\max}) \qquad \frac{t_{\max}}{t_0'} = \frac{1}{2} B\left(\frac{2+\nu}{2}, \frac{1-\nu}{2}\right) = 1.470\ldots$$

$B(p,q)$ = Euler beta function, $B_x(p,q)$ = incomplete beta function

At the limits of time

$$T \to 0 \qquad t \to 0 \qquad a \to t_0'(2+\nu)^{1/\sqrt{3}} \left(\frac{t}{t_0'}\right)^{1/\sqrt{3}}$$

$$T \to \frac{\pi}{2} \qquad t \to t_{\max} \qquad a \to t_0'(1-\nu)^{-1/\sqrt{3}} \left(\frac{t_{\max}-t}{t_0'}\right)^{-1/\sqrt{3}}$$

This is an expanding universe

$$f_T = \frac{da}{adT} = \frac{da}{dt} = \frac{\cos 2T + \sqrt{3}}{\sin 2T} > 0$$

beginning with a big bang $a \sim t^{0.577}$

ending with a big blowup $a \sim (t_{\max} - t)^{-0.577}$.

Since $f_T = \partial_t a$ the Hubble parameter is

$$H(T) = a^{-1}\partial_t a = a^{-1} f_T = \frac{1}{t'_0}(\cos^2 T + \nu)\sin^{-2-\nu} T \cos^{-1+\nu} T$$

and the deceleration parameter is

$$q(T) = \frac{-a\partial_t^2 a}{(\partial_t a)^2} = \frac{-\partial_T f_T}{f_T^2} = \frac{2\left(\sqrt{3}\cos 2T + 1\right)}{\left(\cos 2T + \sqrt{3}\right)^2}$$

The expansion decelerates ($q > 0$, $\partial_T f_T < 0$) until $T_{q=0}$

$$q(T_{q=0}) = 0 \qquad T_{q=0} = \frac{1}{2}\arccos(-1/\sqrt{3}) = 0.6959\,\frac{\pi}{2}$$

then accelerates ($q < 0$, $\partial_T f_T > 0$) until the end.

Interpretation

Fix the current time T_0 by solving $q(T_0) = -0.6$.

Then fix the overall time scale t_0' by solving $H(T_0) = H_0$.

$$t_0 = 0.7 t_H \qquad t_{\max} = 1.6 t_H \qquad a(t_0) = 1.5 t_H$$

where $t_H = H_0^{-1} = 4.6 \times 10^{17}\,\mathrm{s}$ is the Hubble time.

Once the single free parameter t_0' is fixed, the cosmological solution is completely specific.

The fixed point equation $R_{\mu\nu} = \nabla_\mu v_\nu + \nabla_\nu v_\mu$ is equivalent to

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \qquad \kappa = 8\pi G$$

$$\kappa T_{\mu\nu} = \nabla_\mu v_\nu + \nabla_\nu v_\mu - g_{\mu\nu} \nabla_\sigma v^\sigma$$

The source terms on the rhs of the fixed point equation can be interpreted as matter/energy which is (tautologically) dark.

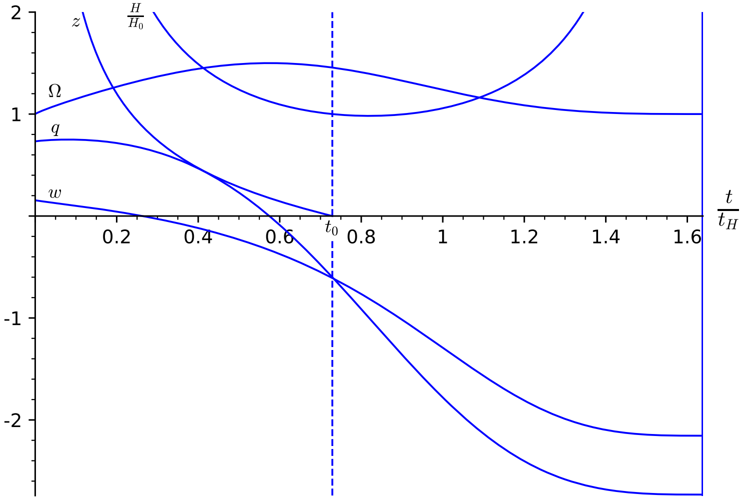
$\mathbf{SO}(4)$-invariance $\implies$ perfect fluid of density $\rho(t)$ and pressure $p(t)$

$$T_{\mu\nu} dx^\mu dx^\nu = \rho(t) dt^2 + p(t) a^2 ds^2_{S^3}$$

$$\rho(t) = \kappa^{-1} a^{-2} \left(3 f_T^2 + 3\right) \qquad p(t) = \kappa^{-1} a^{-2} \left(4 f_T v_T + 3 f_T^2 + 3\right)$$

The equation-of-state and density parameters are

$$w = \frac{p}{\rho} = \frac{\cos 2T}{3\cos 2T + 2\sqrt{3}} \qquad \Omega = \frac{\kappa\rho}{3H^2} = 1 + \left(\frac{\sin 2T}{\cos 2T + \sqrt{3}}\right)^2$$



The cosmological parameters as functions of the co-moving time t. The present time is t_0.

Cosmological parameters at selected values of z

	z	H/H_0	q	w	Ω
$z \gg 1$	$\gg 1$	$0.75\ z^{1.7}$	0.73	0.16	1.0
	1000	1.2×10^5	0.73	0.16	1.0
	100	2.2×10^3	0.73	0.15	1.0
	10	48	0.74	0.15	1.0
$T = \frac{1}{2}T_0$	1.7	4.1	0.74	0.08	1.2
	1.0	2.4	0.69	0.02	1.3
$q = 0$	0.2	1.1	0	-0.3	1.5
$t = t_0$	0	1	-0.6	-0.6	1.5

The point here is just the specificity and the qualitative accord: big bang, then deceleration, then acceleration.

Only $g_{\mu\nu}(x)$ is included in the calculation so there is not much point in comparing quantitatively to real cosmology.

Analysis of the ode.

There is a constant of motion

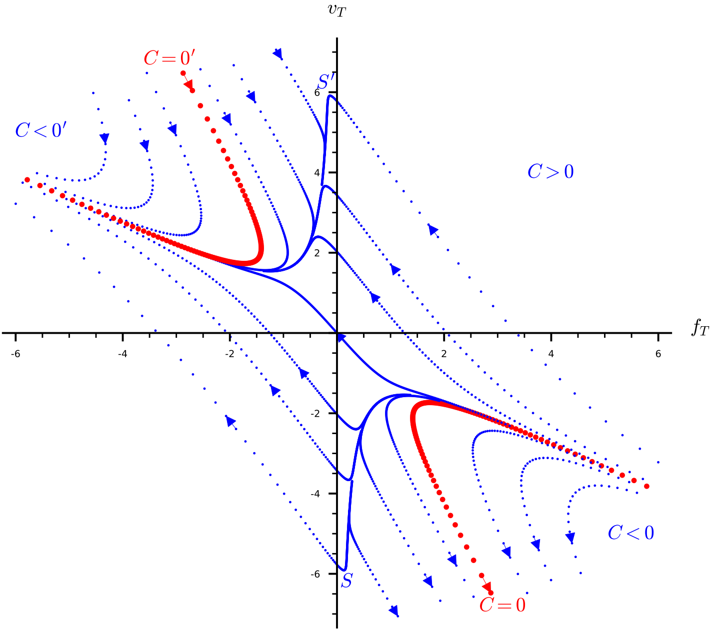
$$C = \frac{1}{a^2}\left(\frac{2}{3}v_T^2 + 2f_T v_T + f_T^2 + 1\right) \qquad \partial_T C = 0$$

The analytic cosmological solution above is the $C = 0$ solution.

Next is the phase portrait of the ode. The trajectories are the solutions. The arrows point in the direction of increasing T. The interval between points in the trajectories is $\Delta T = 0.01$.

The universe is expanding when $f_T = \partial_t a$ is positive. The expansion is accelerating when f_T is increasing.

The two red trajectories are the $C = 0$ solutions. The one in the lower right quadrant labeled $C{=}0$ is the cosmological solution. Its time reflection is labeled $C{=}0'$. The separatrix is labeled S. Its time-reflection is labeled S'.



v_T
f_T
$C=0'$
$C<0'$
S'
$C>0$
S
$C<0$
$C=0$
-6
-4
-2
2
4
6

Every $C \neq 0$ trajectory asymptotes to a $C = 0$ trajectory.

Can expand there around the analytic $C = 0$ solution and show that $C \neq 0$ implies that f_T and v_T always contain an irrational power of T, either $(T^2)^{1+\nu}$ or $(T^2)^{-\nu}$.

So a $C \neq 0$ solution cannot be the analytic continuation of an imaginary time solution.

Second exploratory step: add the $\mathbf{SU}(2)$ gauge field $B_\mu(x)$ and the doublet Higgs scalar $\phi(x)$. The action is

$$S = \hbar \int \mathcal{L} \sqrt{-g}\, d^4x$$

$$\mathcal{L} = \frac{1}{2\kappa\hbar} R + \frac{1}{2g_2^2} \mathrm{tr}(F_{\mu\nu} F^{\mu\nu}) - D_\mu \phi^\dagger D^\mu \phi - \lambda_2 \left(\phi^\dagger \phi - \frac{1}{2} v_2^2 \right)^2$$

$$\kappa = 8\pi G \qquad c = 1$$

The coupling counstants are fixed by

$$v_2 \hbar = 251\,\mathrm{GeV} \qquad m_{\mathrm{Higgs}} = (2\lambda_2)^{1/2} v_2 \hbar = 126\,\mathrm{GeV}$$

$$m_W = \frac{1}{2} g_2 v_2 \hbar = 80.4\,\mathrm{GeV}$$

giving

$$v_2 = \left(2.62 \times 10^{-27}\,\mathrm{s}\right)^{-1} \qquad \lambda_2 = 0.126 \qquad g_2^2 = 0.410$$

The fixed-point equations of the RYMH flow are

$$R_{\mu\nu} - \kappa T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \kappa T^{\sigma}_{\sigma} = \nabla_\mu v_\nu + \nabla_\mu v_\mu$$

$$-\frac{1}{2} D^{\nu} F_{\nu\mu} + \frac{g_2^2}{4} \left(D_\mu \phi \phi^\dagger - \phi D_\mu \phi^\dagger \right) = -v^{\nu} F_{\mu\nu} - D_\mu v_{\mathrm{gauge}}$$

$$-\frac{1}{2} D_\mu D^\mu \phi - \frac{1}{2} \lambda_2 v_2^2 \phi + \lambda_2 (\phi^\dagger \phi) \phi = v^\mu D_\mu \phi + v_{\mathrm{gauge}} \phi$$

lhs = ordinary classical field theory equations of motion

rhs = gauge transformations of the fields $\qquad v_{\mathrm{gauge}}(x) \in \mathbf{su}(2)$

energy-momentum $\qquad T_{\mu\nu} = T^{\mathrm{gauge}}_{\mu\nu} + T^{\mathrm{scalar}}_{\mu\nu}$

$$T^{\mathrm{gauge}}_{\mu\nu} = \frac{\hbar}{2 g_2^2} \,\mathrm{tr}\left(-4 F_{\mu\sigma} F_{\nu}{}^{\sigma} + g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}\right)$$

$$T^{\mathrm{scalar}}_{\mu\nu} = \hbar \left[2 D_\mu \phi^\dagger D_\nu \phi - g_{\mu\nu} D_\sigma \phi^\dagger D^\sigma \phi - g_{\mu\nu} \lambda_2 \left(\phi^\dagger \phi - \frac{1}{2} v_2^2 \right)^2 \right]$$

Assume $\mathbf{SO}(4)$ symmetry or, rather, $\mathbf{Spin}(4)$.

The only rank 2 $\mathbf{SU}(2)$ vector bundle over S^3 is $S^3 \times \mathbb{C}^2$.

Identify the unit 3-sphere S^3 with $\mathbf{SU}(2)$ by

$$\hat{x} \in S^3 \subset \mathbb{R}^4 \quad \longleftrightarrow \quad g_{\hat{x}} = \hat{x}^4 \mathbf{1} + \hat{x}^i u_i \in \mathbf{SU}(2)$$

$\mathbf{Spin}(4) = \mathbf{SU}(2)_L \times \mathbf{SU}(2)_R$ acts on S^3 by

$$U = (g_L, g_R) \qquad g_{\hat{x}} \mapsto g_{U\hat{x}} = g_L g_{\hat{x}} g_R^{-1}$$

and on $\phi(x) \in S^3 \times \mathbb{C}^2$ by

$$\phi(g_{\hat{x}}) \mapsto g_L \phi(g_{U\hat{x}})$$

Assume invariance under this $\mathbf{Spin}(4)$ as an ansatz for solving the equations of motion (to be justified eventually by 2d RG stability).

$\phi(x)$ transforms the same was a spinor does. This is not to say that $\phi(x)$ *is* a spinor. This is to be at early time. The usual $\mathbf{SO}(4)$ symmetry should emerge at late time.

Assuming $\mathbf{Spin}(4)$ invariance

$$g_{\mu\nu}dx^\mu dx^\nu = a(T)^2\left(-dT^2 + ds^2_{S^3}\right)$$

$$\phi(x) = 0$$

$$B_0(x) = 0 \qquad B_i(x) = \frac{1}{2}[1+b(T)]\gamma_i(x) \qquad \gamma_i(x) = g_{\hat{x}}\partial_i g^\dagger_{\hat{x}}$$

$$v^\mu(x)\partial_\mu = v^T(T)\partial_T \qquad v_{\text{gauge}}(x) = 0 \in \mathbf{su}(2)$$

$\gamma_i(x)$ is the unique invariant $\mathbf{su}(2)$-valued 1-form.

Cosmology is now described by three real functions of time

$$a(T) \quad v^T(T) \quad b(T)$$

parity + a gauge transformation $\phi(T,\hat{x}) \to g_{\hat{x}}\phi(T,P\hat{x})$ takes

$$b(T) \to -b(T)$$

$b(T)$ is P odd (and C even) so CP odd.

$\mathbf{Spin}(4)$-invariance $\implies T_{\mu\nu}$ is perfect fluid

$$T_{\mu\nu} dx^\mu dx^\nu = a^2 \rho(T) dT^2 + a^2 p(T) ds^2_{S^3}$$

$$\rho = \frac{3\hbar\,\mathcal{H}_b}{g_2^2\,a^4} + \mathcal{E}_0 \qquad p = \frac{\hbar\,\mathcal{H}_b}{g_2^2\,a^4} - \mathcal{E}_0$$

$\mathcal{E}_0$ is the vacuum energy of the Higgs field at $\phi = 0$.

$$\mathcal{E}_0 = \frac{1}{4}\hbar\lambda_2 v_2^4$$

and the quantity $\mathcal{H}_b$ in the gauge field energy-momentum is

$$\mathcal{H}_b = \frac{1}{2}(\partial_T b)^2 + \frac{1}{2}(b^2 - 1)^2$$

($\mathcal{H}_b = F_+ F_-$. There are two flat gauge fields $b = \pm 1$ where $\mathcal{H}_b = 0$. Analytically continued to $\tau = iT$, $\mathcal{H}_b = 0$ is the instanton equation.)

The invariant fixed-point equations (again with $f_T = a^{-1}\partial_T a$ and $v_T = a^2 v^T$) are

$$3f_T^2 + 3 - \kappa a^2 \rho = \partial_T v_T - 4 v_T f_T$$

$$-2\partial_T f_T - 1 - f_T^2 - \kappa a^2 p = \partial_T v_T$$

$$\frac{1}{2}\partial_T^2 b + b(b^2 - 1) = -v_T \partial_T b$$

Define a time scale t_I and the associated the dimensionless scale factor $\hat{a}$ and a dimensionless parameter ϵ_w

$$t_\mathrm{I} = \left(\frac{3}{\kappa \mathcal{E}_0}\right)^{\frac{1}{2}} \qquad a = t_\mathrm{I}\hat{a} \qquad \epsilon_\mathrm{w} = \left(\frac{\kappa\hbar}{g_2^2}\frac{1}{t_\mathrm{I}^2}\right)^{\frac{1}{4}}$$

t_I will turn out to be the inflation time scale.

The odes now contain only dimensionless variables T, $\hat{a}$, b, v_T and one parameter ϵ_w.

$$3f_T^2 + 3 - 3\epsilon_\mathrm{w}^4 \mathcal{H}_b \hat{a}^{-2} - 3\hat{a}^2 = \partial_T v_T - 4 v_T f_T$$

$$-2\partial_T f_T - 1 - f_T^2 - \epsilon_\mathrm{w}^4 \mathcal{H}_b \hat{a}^{-2} + 3\hat{a}^2 = \partial_T v_T$$

$$\frac{1}{2}\partial_T^2 b + b(b^2 - 1) = -v_T \partial_T b$$

$$\mathcal{H}_b = \frac{1}{2}(\partial_T b)^2 + \frac{1}{2}(b^2 - 1)^2$$

Define Planck and weak time scales by

$$t_{\mathrm{P}} = (\kappa\hbar)^{\frac{1}{2}} = 2.7 \times 10^{-43}\,\mathrm{s}$$

$$t_{\mathrm{w}} = \frac{\hbar}{(m_{\mathrm{Higgs}} m_W)^{1/2}} = \frac{\hbar}{101\,\mathrm{GeV}} = 6.5 \times 10^{-27}\,\mathrm{s}$$

Then the inflation time t_I is the seesaw scale

$$t_{\mathrm{I}} = \left(\frac{3}{\kappa\mathcal{E}_0}\right)^{1/2} = 1.6\,\frac{t_{\mathrm{w}}^2}{t_{\mathrm{P}}} = 2.5 \times 10^{-10}\,\mathrm{s} \quad (\,= 7.5\,\mathrm{cm})$$

The dimensionless parameter ϵ_{w} is the seesaw factor

$$\epsilon_{\mathrm{w}} = 4.1 \times 10^{-17} \qquad \frac{t_{\mathrm{P}}}{t_{\mathrm{w}}} = 1.0\,\epsilon_{\mathrm{w}} \qquad \frac{t_{\mathrm{w}}}{t_{\mathrm{I}}} = 0.66\,\epsilon_{\mathrm{w}}$$

Inflation in conformal time T is

$$T \to 0^- \qquad a \to \frac{t'_I}{-T}$$

In co-moving time $dt = a dT$ this is

$$T \to -e^{-(t-t_0)/t'_I} \qquad a \to t'_I e^{(t-t_0)/t'_I}$$

$$t'_I = \text{ inflation time-scale}$$

So look for solutions meromorphic at $T = 0$ with $a_T = O(T^{-1})$.

Solve the odes recursively order by order in T (using SageMath).

Result:

$$v_T(T) = 0 + O(T^{11})$$

so almost certainly $v_T = 0$ (and likely provable).

So any analytic solution of the fixed-point equations is a solution of the standard classical field theory equations of motion. All the non-standard sources on the rhs vanish.

$v_T = 0 \implies \partial_T \mathcal{H}_b = 0$ so $\mathcal{H}_b$ is a constant of motion $\mathcal{H}_b = E_b$

With $v_T = 0$ the odes become (after a bit of algebra)

$$\partial_T^2 b + V_b'(b) = 0 \qquad V_b = \frac{1}{2}(b^2 - 1)^2 \qquad \frac{1}{2}(\partial_T b)^2 + V_b = E_b$$

$$\partial_T^2 \hat{a} + V_{\hat{a}}'(b) = 0 \qquad V_{\hat{a}} = -\frac{1}{2}(\hat{a}^4 - \hat{a}^2) \qquad \frac{1}{2}(\partial_T \hat{a})^2 + V_{\hat{a}} = E_{\hat{a}}$$

$$E_{\hat{a}} = \frac{1}{2}\epsilon_{\mathrm{w}}^4 E_b \quad \text{(Wheeler–deWitt equation)}$$

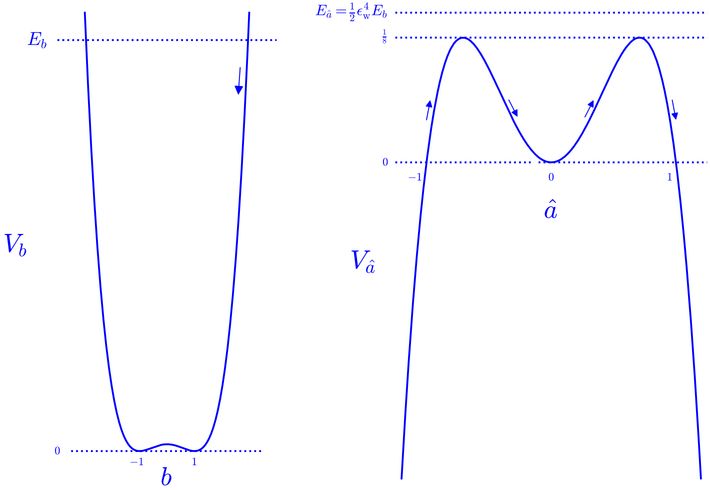
b is an anharmonic oscillator with energy E_b.

$\hat{a}$ is an inverted anharmonic oscillator with energy $E_{\hat{a}}$.

From the above

$$\hat{a} = O(T^{-1}) \implies \hat{a} \to \frac{1}{-T} \qquad \text{so} \qquad a = t_{\mathrm{I}}\hat{a} \to \frac{t_I}{-T}$$

so $t_I = 2.5 \times 10^{-10}\,\mathrm{s}$ is indeed the inflation time scale.



two kinds of inflation:

$$0 < E_{\hat a} \leq \frac{1}{8} \qquad \text{or} \qquad \frac{1}{8} < E_{\hat a} \quad \text{(the choice here)}$$

The classical action in units of $\hbar$ is

$$S = \hbar \int L \, dT$$

$$L = \frac{6\pi^2}{g_2^2} \left(\frac{1}{2} \left[(\partial_T b)^2 - (b^2 - 1)^2 \right] + \epsilon_{\mathrm{w}}^{-4} \left[-(\partial_T \hat{a})^2 + \left(\hat{a}^2 - \hat{a}^4 \right) \right] \right)$$

The classical regime for b is $E_b \gg 1$.

The classical regime for $\hat{a}$ is $\epsilon_{\mathrm{w}}^{-4} E_a \gg 1$.

These are the same since $E_{\hat{a}} = \frac{1}{2} \epsilon_{\mathrm{w}}^4 E_b$.

To check stability of $\phi = 0$ look at the scalar field action density. The quadratic term is

$$(*) = -D_\mu \phi^\dagger D^\mu \phi + \lambda_2 v_2^2 \phi^\dagger \phi$$

$$-D_\mu \phi^\dagger D^\mu \phi = -g^{\mu\nu} [(\partial_\mu + B_\mu)\phi]^\dagger (\partial_\mu + B_\mu)\phi$$

For large $B_i = \frac{1}{2}(1+b)\gamma_i$ this is

$$-\frac{3}{4} b^2 a^{-2} \phi^\dagger \phi + \lambda_2 v_2^2 \phi^\dagger \phi$$

so $\phi = 0$ is stable as long as

$$-\frac{3}{4} b^2 a^{-2} + \lambda_2 v_2^2 < 0$$

which is

$$a^2 < \frac{3}{4} \frac{b^2}{\lambda_2 v_2^2} \approx b^2 t_{\mathrm{w}}^2$$

b spends almost all its cycle at the entremes where $b^2 \approx (2E_b)^{1/2}$ so ϕ is pinned to 0 as long as

$$a^2 < E_b^{1/2} t_{\mathrm{w}}^2$$

The pinning to $\phi = 0$ will end when a inflates to a_{unpin}

$$a_{\mathrm{unpin}} \approx E_b^{1/4} t_{\mathrm{w}} \approx E_b^{1/4} \epsilon_{\mathrm{w}} t_{\mathrm{I}} \approx E_{\hat{a}}^{1/4} t_{\mathrm{I}} \qquad \hat{a}_{\mathrm{unpin}} \approx E_{\hat{a}}^{1/4}$$

Inflation then enters the end stage as ϕ descends to its vev. This remains to be calculated.

To have a significant amount of inflation $E_{\hat{a}}^{1/4}$ must be very large. Maybe this should be discouraging.

B_μ should descend to 0 in the end stage of inflation. The non-zero $b(T)$ may break CP during the descent.

Once ϕ reaches its vev and B_μ reaches 0, the standard $\mathbf{SO}(4)$ symmetry should emerge.

The anharmonic oscillator is solved by elliptic functions. In terms of the doubly periodic Weierstrass elliptic function $\wp(z, \omega_1, \omega_3)$

$$b(T)^2 = \frac{2}{3} - \wp(T - T_0 + \omega_3/2, \omega_1, \omega_3)$$

$$\omega_1 \approx E_b^{-1/4} \qquad \omega_3 \approx iE_b^{-1/4}$$

$b(T)$ is periodic in time and also in imaginary time

$$b(T) = b(T + 2\omega_1) = b(T + 2\omega_3)$$

periodicity in imaginary co-moving time = inverse temperature

$$\frac{\hbar}{kT_b} = a\frac{2\omega_3}{i} \approx aE_b^{-1/4}$$

The background gauge field has temperature T_b?

$$\frac{\hbar}{kT_b} = a\frac{\omega_3}{i} \approx aE_b^{-1/4}$$

$$kT_b \approx \frac{\hbar}{aE_b^{-1/4}} \approx \frac{\hbar}{t_\mathrm{w}}\frac{a_\mathrm{unpin}}{a} \approx 100\,\mathrm{GeV}\ \left(\frac{a_\mathrm{unpin}}{a}\right)$$

How would this background temperature manifest itself?

Rather hot early in the inflation?

energy densities up to unpinning:

$$\mathcal{E}_0 \approx \frac{\hbar}{t_\mathrm{w}^4} \qquad \rho_\mathrm{gauge} \approx \frac{\hbar}{t_\mathrm{w}^4}\left(\frac{a_\mathrm{unpin}}{a}\right)^4$$

Does this version of inflation do what inflation is needed to do?

Next steps:

1. Find an analytic classical solution with $\phi \neq 0$. It should look like the inflationary analytic solution at early time and like the big bang analytic solution at late time.
   Assume $SU(2) \times U(1)$ symmetry with the $U(1)$ acting projectively on ϕ, i.e.,

   $$(1 \times e^{i\theta\sigma_3})\phi = e^{i\theta}\phi \qquad \phi = (\phi_1(T), 0)$$

   Now there are more variables describing the invariant metric and gauge field and the invariant gauge transformations.

2. Figure out how to do microscopic physics in these classical macroscopic backgrounds. Does $b(T)$ impose the temperature T_b on everything that interacts with the gauge field?

   The microscopic physics will provide additional energy-momentum and current in the classical macroscopic equations of motion.

This is a top-down approach to cosmology.

Find principles for a classical calculation of macroscopic cosmology. The calculation should start by explaining basic qualitative features like the big bang, deceleration followed by acceleration, inflation, cosmological CP violation, the origin of cosmological temperature.

Incrementally incorporate more fields of the Standard Model. Incrementally incorporate the microscopic.

Look for principles of calculation that yield specificity: solutions with very few free parameters and explicit calculation of observables.

Stay within the Standard Model. Effective QFT is too vast a parameter space, too unspecific. So far, after 47 years, there is no direct evidence of any particles/fields beyond the Standard Model. Of course we should keep looking. But maybe it is not so crazy to look into other ideas for fundamental physics that might provide testable explanations of what is not explained by the Standard Model QFT.