Lecture 19: Radiative Corrections Nov. 8, 2007

Last time, we calculated that if an electron undergoes an impulse which changes its velocity from $\vec{v}$ to $\vec{v}\,'$, in a frame where its energy doesn't change, the expected number of photons coming off with wave number in the interval $[k, k+\Delta k]$ is

$$\frac{\alpha}{\pi}\frac{\Delta k}{k} I(\vec{v}, \vec{v}\,'),$$

with an angular distribution

$$\propto \frac{2p\cdot p'}{\hat{k}\cdot p'\,\hat{k}\cdot p} - \frac{m^2}{(\hat{k}\cdot p')^2} - \frac{m^2}{(\hat{k}\cdot p)^2}.$$

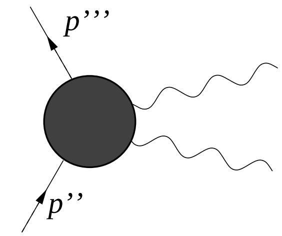
We saw that at high electron energy,

$$I(\vec{v}, \vec{v}\,') \to 2\ln\frac{-q^2}{m^2},$$

where $q = p' - p, q^2 = -\vec{q}^{\,2}$.

More importantly, because the energy emitted in each wavenumber interval Δk is independent of k, the expected number of photons diverges, both at large k, where we should not believe the results because our impulse approximation and the lack of loss of electron energy is unrealistic, but also as $k \to 0$, where our calculation should be correct.

Today we will calculate, to order e^2, the quantum-mechanical probablitiy that a single photon of momentum $\vec{k}$ will be emitted. It is clear that each additional photon emitted takes an extra power of e^2, so in perturbation theory the probability is considered small even if it is multiplied by a large constant. Thus if we find a probability greater than one, it doesn't mean a mistake in our calculation itself, but just that this order in perturbation theory will be swamped by higher order terms. We will need a better way to ask our questions perturbatively than to ask what the probability that exactly one (or exactly zero) photons are emitted.



Some more comments:
In 6.20, note that $\mathcal{M}_0(p''', p'')$ is a matrix in spinor space.

In 6.36 we use

$$\gamma^0\gamma^i = \begin{pmatrix} -\sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$$

At the bottom of page 187 we are using

$$\begin{aligned} \bar{u}\sigma^{i0}u &= -iu^\dagger \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} u \sim 0 \\ \bar{u}\sigma^{ij}u &= \epsilon_{ijk}u^\dagger \begin{pmatrix} 0 & \sigma^k \\ \sigma^k & 0 \end{pmatrix} u \sim 2m\epsilon_{ijk}\xi^\dagger\sigma^k\xi. \end{aligned}$$

On page 188, note that if the classical Maxwell field

$$A^{\rm cl}(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x}\tilde{A}(k),$$

then

$$\widetilde{\nabla\times A}(k) = -i\vec{k}\times\tilde{\vec{A}}(k).$$

Read Peskin and Schroeder pp. 182–189, and also "Schwinger Trick and Feynman Parameters", from the supplementary material posted on the web.