Lecture 19: Nov. 7, 2013
Quantum Bremstrahlung; Form Factors
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Last time, we calculated that if an electron undergoes an impulse which changes its velocity from $\vec{v}$ to $\vec{v}'$, in a frame where its energy doesn’t change, the expected number of photons coming off with wave number in the interval $[k, k + \Delta k]$ is

$$\frac{\alpha}{\pi} \frac{\Delta k}{k} I(\vec{v}, \vec{v}'),$$

with an angular distribution

$$\propto \frac{2p \cdot p'}{k \cdot p' k \cdot p} - \frac{m^2}{(k \cdot p')^2} - \frac{m^2}{(k \cdot p)^2}.$$ We saw that at high electron energy,

$$I(\vec{v}, \vec{v}') \rightarrow 2 \ln \frac{-q^2}{m^2},$$

where $q = p' - p$, $q^2 = -\bar{q}^2$.

More importantly, because the energy emitted in each wavenumber interval $\Delta k$ is independent of $k$, the expected number of photons diverges, both at large $k$, where we should not believe the results because our impulse approximation and the lack of loss of electron energy is unrealistic, but also as $k \rightarrow 0$, where our calculation should be correct.

Today we will calculate, to order $e^2$, the quantum-mechanical probability that a single photon of momentum $\vec{k}$ will be emitted. It is clear that each additional photon emitted takes an extra power of $e^2$, so in perturbation theory the probability is considered small even if it is multiplied by a large constant. Thus if we find a probability greater than one, it doesn’t mean a mistake in our calculation itself, but just that this order in perturbation theory will be swamped by higher order terms. We will need a better way to ask our questions perturbatively than to ask what the probability that exactly one (or exactly zero) photons are emitted.
Some more comments:
In 6.20, note that $\mathcal{M}_0(p''', p'')$ is a matrix in spinor space.

Just below 6.36 we use
\[ \gamma^0 \gamma^i = \begin{pmatrix} -\sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} \]

At the bottom of page 187 we are using
\[
\bar{u}\sigma^0 u = -iu^\dagger \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} u \sim 0
\]
\[
\bar{u}\sigma^{ij} u = \epsilon_{ijk} u^\dagger \begin{pmatrix} 0 & \sigma^k \\ \sigma^k & 0 \end{pmatrix} u \sim 2m\epsilon_{ijk}\xi^\dagger \sigma^k \xi.
\]

On page 188, note that if the classical Maxwell field
\[ A^{\text{cl}}(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \tilde{A}(k), \]
then
\[ \nabla \times A(k) = -i\vec{k} \times \tilde{A}(k). \]

Read Peskin and Schroeder pp. 182–189, and also “Schwinger Trick and Feynman Parameters”, from the supplementary material posted on the web.