What about the weak interactions?

- Discovery of radioactivity - 1896 Becquerel Uranium
  - $N_1 \rightarrow N_2 + \beta^-$
  - $\rightarrow$ electron

1910 - 1930 realized $\beta$ spectrum in $\beta$ decay is continuous!

How is this consistent w/ energy conservation??

Maybe energy is not conserved? (Bohr)

Pauli, Fermi: neutrinos!

$N_1 \rightarrow N_2 + \beta^- + \nu < \text{(neutral)}$

then $\beta$ energy not unique
\[ n \rightarrow p + e + \bar{\nu}_e \]

1933 Fermi landmark paper on weak interactions & \( p \)-decay
- also established formalism for matter creation/decay
  - “Fermi golden rule”
- established scale of weak interactions
- \( G_F \) \( \bar{p} \bar{e} \nu_e \) operator mediates \( p \)-decay \( \rightarrow \) beginning of effective field theory

“Fermi constant” \( G_F \sim \left( \frac{1}{100 \text{ GeV}} \right)^2 \)
- scale of weak interactions \( \sim 100 \text{ GeV} \)
- (compare \( q \rightarrow q \) \( \rightarrow 100 \text{ MeV} \)
- mass of \( W \) & \( Z \) bosons that mediate weak force.
SM: \( G \& M \rightarrow \text{QCD} \)

U(1) gauge theory

Strong nuclear force \( \rightarrow \text{QCD} \)

SU(3) gauge theory


Radioactive decay \( \rightarrow \text{QCD} \)

Electroweak theory

SU(2) \& U(1) gauge theory

\( \downarrow \) Spontaneous symmetry

Quarks & leptons

Higgs boson


+ Flavor

Cosmic rays \( \rightarrow \) discovery of muon \( 10 \text{ MeV} \)


Very heavy cousin of electron! "Who ordered that?"

Unstable: \( \mu \rightarrow e + \nu_e + \bar{\nu}_e \) (like p decay)
Later \( \tau \) \( m_\tau = 1.7 \text{ GeV} \) even heavier & more unstable version of electron

\[
\begin{pmatrix}
e^\tau \\
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
\]

same interactions under SM.

Also quarks

\[
\begin{pmatrix}
d \\
\upsilon \\
s \\
t
\end{pmatrix}
\]

also 3 generations!

why ?? \( \rightarrow \) nobody knows

one special thing about 3 generations \( \text{vs 2} \) \( \rightarrow \) starting \( \frac{1}{3} \),

CKM matrix observed in Nature \( \rightarrow \) CP violation is allowed.
Unit I: Introduction of AEO

1. E&M as a relativistic Lagrangian field theory (classical)

Recall:
\[ \nabla \cdot B = 0 \]
\[ \nabla \times E + \frac{\partial B}{\partial t} = 0 \]
\[ \nabla \cdot E = 0 \]
\[ \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = 0 \]

Maxwell's eqns in vacuum

\[ A \text{ potential formalism} \]
\[ (\phi, \mathbf{A}) \]
\[ \mathbf{B} = \nabla \times \mathbf{A} \]
\[ \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \]

Gauge invariance:
\[ \mathbf{E} & \mathbf{B} \text{ unchanged for any } \phi \]
\[ \mathbf{A} \rightarrow \mathbf{A} - \nabla \phi \]
\[ \phi \rightarrow \phi - \phi \]
Comment about "gauge symmetry" $\rightarrow$ "local symmetry" $\partial (x,t)$

different than ordinary "global symmetry"

- physical laws are unchanged
- global symmetry transformation relates distinct states of system

redundancy of description

physical laws unchanged + states related by gauge symmetry are identified

$\leftarrow$ there is no circle "mod out" by gauge transformations but not global symmetries.
Maxwell \rightarrow \text{pot'l}

\text{Coulomb gauge } \quad \nabla \cdot E = 0 \quad \rightarrow \quad \nabla^2 A - \frac{\partial}{\partial t} A = 0.

\text{plan wave eqn}
\quad \vec{A} \sim e^{-i(k \cdot x - \omega t)}
\quad \text{massless, relativistic dispersion relation}

A_{\mu} = (-\mathbf{E}, \mathbf{A})

A_{\mu} \rightarrow A_{\mu} - \partial_{\mu} y \quad \text{gauge transf.}

F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \quad \text{gauge invariant field strength.}

\text{check:} \quad F_{0i} = \nabla_i A_0 - \nabla_0 A_i = -\varepsilon_i \quad F_{ij} = \frac{\partial A_j - \partial A_i}{\varepsilon_{ijk}} = \varepsilon_{ijk} B_k
(HW1) check: Maxwell's eqns \[ \nabla^2 F_{\mu\nu} = 0 \]

Reformulate w/ Lagrangians & Action principle

\[ S = \int dt \ L = \int dt \ d^3 x \ L \]

\[ = \int d^4 x \ L \]

for Maxwell they

\[ \text{want } L : \text{Lorentz+gauge} \]

\[ \text{require } L \text{ to be Lorentz invar\textsuperscript{+}\textcircled{4} if w\textsuperscript{+}\textsuperscript{4} system to be Lorentz invar\textsuperscript{+}\textcircled{4}.} \]
unique object Lorentz & gauge in 4
quadratic in $A_m$
\[ L = \frac{1}{4} \varepsilon_{mn} F^m \cdot F^n \]

\[ \varepsilon \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \]

\( L = (\partial A^\mu - \partial^\nu A^\nu)(\partial A^\nu - \partial^\mu A^\mu) \quad \text{total derivative} \)

\[ \partial^\mu F^\mu = 0 \]

Equations of motion

\[ \partial^\mu \left( \varepsilon^{\mu\nu} \frac{\partial}{\partial \partial A^\nu} \right) = 0 \]

\[ \varepsilon^{\mu\nu} \partial \partial A^\nu = 0 \]

\( \partial^\nu A^\nu = \varepsilon^\nu(\partial A^\nu) \; \text{set to zero w/ gauge choice} \)

\( \text{e.g., C. gauge or Lorentz gauge} \)
\[ \Box A_\nu \equiv \nabla_\nu A_\nu = 0 \]

Lorentz in\textsuperscript{\textit{f}} wave equation

\[ \frac{\partial^2}{\partial t^2} - \nabla^2 \]

“D'Alembertian operator”

another covariant derivative

that \( 2\Box A_\nu = 0 \equiv \text{Maxwell’s eqns.} \)