The term "Strange Metal" is used to describe the unusual metallic behavior, epitomized by the normal state of cuprate high temperature superconductors, in which the resistivity is linear in temperature over a wide range of temperature,

\[ \sigma = \sigma_0 + AT \]

where the residual resistivity \( \sigma_0 \), is a small fraction of the linear rise, and in which the "scattering time", determined from a fit to the Daido formula

\[ \tau = \frac{m}{ne^2 \tau} \tag{1} \]

appears to be of order the "Planck time"

\[ Z_{\alpha} = \sum_{\text{histories}} e^{-S/k} \]

\[ \gamma_p \sim \frac{\hbar}{k_B T}, \quad \Delta t \sim \hbar \frac{\Delta E}{k_B T} \tag{2} \]
that is
\[ \rho \approx \frac{m}{ne^2} \left( \frac{k_B T}{\hbar} \right) \]

What is so strange about this? After all, isn't the resistivity of copper linear in temperature? Well—
it we look at conventional metals, they do show
linear resistivity over limited regions of temperature,
intact the scattering rate of electrons induced by
scattering off phonons, is linear in conventional
metals
\[ \frac{\hbar}{T_{tr}} = \frac{2\pi \lambda}{\alpha} \frac{k_B T}{\lambda} \]

where \( \lambda \) is the electron-phonon coupling constant,
however such behavior usually extends over a very
limited intermediate scale, and does not extend to
either low temperatures (where $\tau_{tr} \propto T^5$) or high temperatures, where the scattering tends to saturate.

Moreover, in conventional "Fermi liquids", the low temperature resistivity is governed by electron-electron scattering for which

$$\frac{1}{\tau_{tr}} \propto \left( \frac{T}{T_F} \right)^2$$

where $T_F$ is the Fermi temperature.

None of these features are seen in strange metals; indeed at optimal doping, the linear resistance of cuprate superconductors extends from their superconducting transition temperature, $T_c$, up to $1000k$. 

None of
Moreover, strange metallic behavior is also seen in certain heavy electron materials, such as CeCoIn5, in iron based high temperature superconductors, and most recently, in twisted-bilayer graphene. These are metals with widely differing electronic structure, yet their commonalities suggest a uniform underlying explanation.
BaFe$_{2}$As$_{2-x}$P$_x$ ($x=0.31$)


Iron tetrahedron

As, P, S, Se, Te

Candidate Strange Metal
In shock, in 2020, some 30 odd years after they were first discovered, Strange metals remain a major vexed problem in condensed matter physics, and they make us believe that there is a new kind of metal, a counterpart to the well-established Landau Fermi liquid, that we have yet to understand.
In the next three lectures we will explore the phenomenology and the current state of understanding of strange metals. The outline of the lectures is:

2. The Marginal Fermi Liquid Phenomenology.
3. The Ioffe-Larkin Model.
4. A survey of current theories and experiments.
**Conventional Metals**

**Drude**

\[ \sigma_{xx} \sim \frac{n e^2 \tau_{tr}}{m} \]
\[ \sigma_{xy} \sim \sigma_{xx} \left( \frac{\omega_c T}{\Theta_H} \right) \]
\[ \Delta \sigma_{xx} \sim \sigma_{xx} \left( \frac{\omega_c T}{\Theta_H} \right)^2 \]

\[ \Theta_H \]

**Diffusion**

\[ V_D \sim V_F T \]

**Quasiparticle**

\[ z \sim \left( 1 - \frac{\partial z}{\partial u} \right)^{-1} A(k, \omega) \]

**Landau Fermi Liquid**

\[ \tau^{-1} \sim \frac{\omega^2 + (2\pi T)^2}{E_F} \]

\[ \varepsilon = \varepsilon(T, B) - \varepsilon(T, 0) \]

\[ g = g(T, 0) \]

\[ \frac{\delta g}{g} \propto (\omega_c T)^2 \propto \left( \frac{B}{g} \right)^2 \]

**Kohler's Rule**

**One Relaxation Time**

**Cancellation because** \( T_H = T \)
Strange Metals

Violate many of these properties

\[ T^{-1} = \left( \frac{k_B T}{\hbar} \right)^\eta \]

\( \eta = 1 \)

Quasiparticles not well defined.

"Marginal Fermi Liquid"

"Non Fermi Liquid"

"Plankian Dissipation"

Violate Kohler's Rule

\[ \tau_H^{-1} = \frac{T^2}{W} + b \]

Origin of a Modified Kohler's Rule

Two relaxation times (at each point on the FS?)

Impurities
Resistivity of La$_{1.825}$Sr$_{0.175}$CuO$_4$ and YBa$_2$Cu$_3$O$_7$ to 1100 K: Absence of Saturation and Its Implications

M. Gurvitch and A. T. Fiory
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 29 July 1987)

\[ (\sigma = \frac{\mu e^2}{4\pi} \tau) \]

\[ \sigma = \frac{ne^2}{\tau} \tau = (\varepsilon_0 \omega_p^2) \tau \]

\[ \tau^{-1} = 2\pi \lambda \left( \frac{k_B T}{\hbar} \right) = \eta \left( \frac{k_B T}{\hbar} \right) \]
<table>
<thead>
<tr>
<th>Material</th>
<th>( \lambda )</th>
<th>( \eta = 2\pi \lambda )</th>
<th>( b = \frac{C_{\text{mfp}}}{a} ) (300K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LaSCO</td>
<td>0.1</td>
<td>0.6</td>
<td>7.6</td>
</tr>
<tr>
<td>YBCO</td>
<td>0.3</td>
<td>1.8</td>
<td>5.5</td>
</tr>
<tr>
<td>( V_3 \text{Si} )</td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

- No saturation despite closeness to Lofte Regel.
- Rules out an e- -phonon mechanism for S.C.
Hall constant is strongly $T$-dependent, and behaves strangely under Zn doping.

- $\cot \Theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{1}{\omega_e \tau_H} \sim T^2 + c_i$

- $\tau_H^{-1} \sim \left(\frac{e\theta T}{W}\right)^2 + b_i \quad W \sim 800 K$

Do not obey Köhlers Rule.
If \( \Delta \sigma_{xx} \sim \sigma_{xx} (\omega_c T_H)^2 \)

\[
\left( \frac{\sigma_{xy}}{\sigma_{xx}} \right) \sim (\omega_c T_H) \\
\tan \Theta_H \sim (\sigma_{xy} \sigma_{xx})^2 \\
\rho_{xx} \sim \frac{\partial \rho_{xx}}{\partial T} \Rightarrow \rho = \frac{\Delta g_{xx}}{\sigma_{xx}^2 \sigma_{xy}^3} = \text{const.}
\]

**Normal-state Hall Angle and Magnetoresistance in quasi-2D Heavy Fermion CeCoIn_5 near a Quantum Critical Point**

Y. Nakajima\(^1\), K. Izawa\(^1\), Y. Matsuda\(^1\), S. Uji\(^2\), T. Terashima\(^2\), H. Shishido\(^3\), R. Settai\(^3\), and Y. Onuki\(^3\), and H. Kontani\(^4\)

**Similar Behavior in CeCoIn_5**

\[
\sigma_{xy} = \frac{n e^2}{m} \frac{1}{T_{tr} T_H}
\]
Marginal Fermi Liquid Theory

Phenomenology of the Normal State of Cu-O High-Temperature Superconductors

C. M. Varma, P. B. Littlewood, and S. Schmitt-Rink
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

E. Abrahams and A. E. Ruckenstein
Sinan Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855
(Received 7 August 1989)

\[ P(\tilde{q}, \omega) = \begin{cases} \frac{(\omega/T)}{\omega - i\pi/2} & |\omega| \ll T \\ \text{sgn}(\omega) & |\omega| \gg T \end{cases} \]

\[ \text{Charge or spin polarizability:} \]

\[ P''(\omega) \sim \tanh\left(\frac{\omega}{T}\right) \]

\[ P(\omega) \sim \int_{-\infty}^{\infty} \frac{d\nu}{\pi} \frac{P''(\nu)}{\omega - \nu} \sim \ln \omega \]

\[ x = \max(|\omega|, T) \]

\[ \Sigma(\tilde{q}, \omega) \sim \frac{\lambda}{g N(0)^2} \left[ \omega \ln \frac{x}{\omega} - i \frac{\pi}{2} x \right] \]

\[ \Rightarrow \Sigma''(\tilde{q}, \omega) = \Gamma = \frac{\pi}{2} \lambda \begin{cases} |\omega| & \omega > T \\ T - \frac{\pi}{2} & \omega < T \end{cases} \]
Quasiparticles are no longer well defined. $\frac{\Gamma}{|\epsilon_k|} = \text{constant.}$

$$G(k, \omega) = \frac{1}{\omega - \epsilon_k - \Sigma(\omega)} = \frac{1}{\omega - \epsilon_k - \lambda(\omega \ln \frac{\omega}{\omega_c} + \frac{i\pi}{2} \omega)}$$

$A(\omega) = G(\omega - \delta)''/\pi$

RAMAN SCATTERING

$$I(\omega) \propto -\int [1 + n(\omega)] \text{Im} \epsilon^2(\omega, \omega)$$

$$\sim \int [1 + n(\omega)] \text{Im} P(\omega)$$

$$\leq \begin{cases} \frac{\Gamma_{\omega}}{\omega} \text{Im} P(\omega) & \omega < \gamma \\ \text{Im} P(\omega) & \omega > \gamma. \end{cases}$$
ELECTRON ENERGY LOSS SPECTROSCOPY

\[ \chi(q, \omega) = \frac{P(q, \omega)}{1 - e^{-\omega / \omega_c}} V(q) \, P(q, \omega) \]

At large \( q \), \( \chi \sim P \).
\( V(q) \sim \frac{e^{-qR}}{q} \) is fit from data.

\((\eta \equiv P)\)
Essentially local (\( q \) independent) MFL particle-hole continuum.

"Local criticality"

\[ \xi_\pi \sim \frac{\hbar}{k_B T} \]

\[ \xi_e = \xi_\pi \]

\( \varepsilon \) = dynamical critical exp.

An energy scale is seen to develop in the overdoped samples.

\[ \xi_e \sim \ln \xi_\pi \sim \xi_\pi \]

\( \Rightarrow \varepsilon = \infty! \)

"Local quantum criticality"
What is the microscopic origin of $P(\omega) \sim \text{const}$?

Helpful to look in the time domain

\[ \text{FL} \quad \begin{array}{c}
\alpha \leftarrow \frac{1}{\tau^2} \\
G(\tau) \sim 1/\tau
\end{array} \quad \Leftrightarrow \quad \Sigma''(\omega) \sim \int \frac{1}{\tau^2} e^{-i\omega \tau} d\tau \sim \omega \]

\[ \text{MFL} \quad \begin{array}{c}
\alpha \leftarrow \frac{1}{\tau^2} \\
S_g(\tau)
\end{array} \quad \Leftrightarrow \quad \Sigma''(\omega) \sim \int \frac{1}{\tau^2} e^{-i\omega \tau} d\tau \sim \omega
\]

This is one way to achieve a MFL

seen in the two-channel Kondo model

but what would stabilize a local fermion in the lattice?

"local quantum criticality".