Outline of the Topics

1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
4. Large N expansion for the Kondo Lattice
5. Heavy Fermion Superconductivity
6. Topological Kondo Insulators
7. Co-existing magnetism and the Kondo Effect.

Please ask questions!
Glue vs Fabric.
Glue vs Fabric.

Glue: Spin fluctuations = pairing bosons

Eliashberg Approach (cf B. Keimer et al)
Glue vs Fabric.

Glue

Spin fluctuations = pairing bosons

Eliashberg Approach (cf B. Keimer et al)
Glue vs Fabric.

Glue

Spin fluctuations = pairing bosons

Fabric: spins **make** the pairs

$(\pi, 0) \rightarrow s^\pm$
Glue vs Fabric.

**Glue**
Spin fluctuations = pairing bosons

(\pi, 0) \rightarrow s^{\pm}

Eliashberg Approach (cf B. Keimer et al)

**Fabric:** spins *make* the pairs


Emery & Kivelson: composite pairs (1993)

\[ \text{NpPd}_2\text{Al}_2 \]

\[ C/T \text{ vs. temperature (K)} \]

\[ C_p/T \]
Glue vs Fabric.

**Glue**  
Spin fluctuations = pairing bosons

![Diagram of Glue mechanism](image)

Eliashberg Approach (cf B. Keimer et al)

**Fabric:** spins *make* the pairs

Emery & Kivelson: composite pairs (1993)

![Diagram of Fabric mechanism](image)

\[ R \ln W = \int_0^{T'} dT' \frac{C''}{T'} \]

“Hilbert Space Spectroscopy”
Glue vs Fabric.

Glue

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\( \sim 1/3 \ R \ln(2) \)

\[
R \ln W = \int_0^T dT' \frac{C''}{T'}
\]

“Hilbert Space Spectroscopy”

SPIN Hilbert space BUILDS the pairs.
Glue vs Fabric.

**Glue**

Spin fluctuations = pairing bosons

\[ (\pi,0) \rightarrow s^\pm \]

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**Fabric: spins** *make* the pairs


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\[ \sim 1/3 \, R \, \ln(2) \]

~1/3 R ln(2)

\[
R \ln W = \int_0^{T'} dT' \frac{C''}{T'}
\]

“Hilbert Space Spectroscopy”

SPIN Hilbert space BUILDS the pairs.
**UPt₃**

Pauli paramagnetic by 30K

$T_c = 0.5K$


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**CeCoIn₅**

No Pauli paramagnetism

$T_c = 2.3K$

Shishido et al. JPSJ 71, 162 (2002)
Heavy Fermions and the Kondo Lattice

These materials, the entropy of condensation

\[ S_c = Z T_c \]

can be as large as \( \frac{1}{3} R \ln 2 \) per rare earth ion, indicating that the spin is, in some way, entangled with the conduction electrons to build the condensate. In this situation, we need to be able to consider the Kondo effect and superconductivity on an equal footing.

Fig. 12: (a) Phase diagram of 115 compounds CeMIn\(_5\), adapted from [43], showing magnetic and superconducting phases as a function of alloy concentration. (b) Sketch of specific heat coefficient of CeCoIn\(_5\), (with nuclear Schottky contribution subtracted), showing the large entropy of condensation associated with the superconducting state. (After Petrovic et al 2001 [39]).

4.1 Symplectic spins and SP (N).

Although the SU(N) large \( N \) expansion provides a very useful description of the normal state of heavy fermion metals and Kondo insulators, there is strangely, no superconducting solution. This shortcoming lies in the very structure of the SU(N) group. SU(N) is perfectly tailored to particle physics, where the physical excitations - the mesons and baryons appear as color singlets, with the meson a a quark-antiquark singlet while the baryon is an \( N \)-quark singlet, (where of course \( N = 3 \) in reality). In electronic condensed matter, the meson becomes a particle-hole pair, but there are no two-particle singlets in SU(N) beyond \( N = 2 \). The origin of this failure can be traced back to the absence of a consistent definition of time-reversal symmetry in SU(N) for \( N > 2 \). This means that singlet Cooper pairs and superconductivity cannot develop at the large \( N \) limit.

A solution to this problem which grew out an approach developed by Read and Sachdev [44] for frustrated magnetism, is to use the symplectic group \( \text{SP}(N) \), where \( N \) must be an even number.
\[ \sigma \in \left( -\frac{1}{2}, \frac{1}{2} \right) \rightarrow \left( -\frac{N}{2}, \frac{N}{2} \right) \]
\[ H = \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha} + \frac{J_K}{N} \sum_j c_{j\alpha}^\dagger c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j) \]
\[ H = \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha} + \frac{J_K}{N} \sum_j c_{j\alpha}^\dagger c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j) \]
\( \mathbf{SU}(N): \)

**Mesons**

\( \bar{q}q \)

**Baryons**

\( q_1 q_2 \ldots q_N \)

\[
H = \sum_{\mathbf{k}\alpha} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha} + \frac{J_K}{N} \sum_{j} c_{j\alpha}^\dagger c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j)
\]

\( N \to \infty \)

\[
S[\psi]
\]

\[
\tau, x
\]

\[
\psi(x, \tau)
\]

\[
\frac{1}{N} \sim \hbar_{\text{eff}}
\]

?
\[ H = \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha} + \frac{J_K}{N} \sum_j c_{j\alpha}^\dagger c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j) \]
**SU(N):**

Mesons: $\bar{q}q$

Baryons: $q_1 q_2 \cdots q_N$

No Pairs!

**SP(N):**

Mesons: $\bar{q}q$

Baryons: Cooper pairs: $q_\alpha q_{-\alpha}$

$H = \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha} + \frac{J_K}{N} \sum_j c_{j\alpha}^\dagger c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j)$

$\frac{1}{N} \sim \hbar_{\text{eff}}$

$N \to \infty$

$S[\psi]$

$\tau, x$

$\psi(x, \tau)$

Cooper pairs
\[ S_{\alpha\beta} = f^\dagger_{\alpha} f_{\beta} - \text{sgn}(\alpha\beta) f^\dagger_{-\beta} f_{-\alpha} \]

\[ H = \sum_{k\alpha} \epsilon_k c^\dagger_{k\alpha} c_{k\alpha} + \frac{J_K}{N} \sum_j c^\dagger_{j\alpha} c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j) \]

- **SU(N):** Mesons
  \[ \bar{q}q \]
  No Pairs!

- **SP(N):** Cooper pairs
  \[ q_1 q_2 \cdots q_N \]

\[ \frac{1}{N} \sim \hbar_{\text{eff}} \]

[Diagram showing classical path and relation between SU(N) and SP(N) with N \to \infty]
SP(N) Large N Approach.

\[ H = H_c + H_K + H_{RKKY} \]

\[
H_K = \frac{J_K}{N} \sum_j c^\dagger_j a_c j_\beta S_\beta \alpha(j) \rightarrow -\frac{J_K}{N} \sum_{i,j} \left( (c^\dagger_j a_c j_\beta)(f^\dagger_j c j_\beta) + \bar{\alpha} \bar{\beta} (c^\dagger_j a_c f^\dagger j_{-\alpha})(f_{j-\beta} c j_\beta) \right)
\]

\[
H_M = \frac{J_H}{2N} \sum_{(i,j)} S_\alpha \beta(j) S_\beta \alpha(j) \rightarrow -\frac{J_H}{N} \sum_j \left( (f^\dagger i a_c f j_\alpha)(f^\dagger j_\beta f j_\beta) + \bar{\alpha} \bar{\beta} (f^\dagger i a_c f^\dagger j_{-\alpha})(f_{j-\beta} f j_\beta) \right)
\]

Uniform solution:

\[
H = \sum_{k,\alpha > 0} (\bar{c}_{k\alpha}^\dagger, \bar{f}_{k\alpha}^\dagger) \left( \epsilon_k \tau_3 \begin{pmatrix} V \tau_3 & V \tau_3 \end{pmatrix} \begin{pmatrix} \bar{c}_{k\alpha} \bar{f}_{k\alpha} \end{pmatrix} + N s N \left( \frac{|V|^2}{J_K} + 2 \frac{\Delta^2 H}{J_H} \right) \right)
\]
SP(N) Large N Approach.  

\[ H = H_c + H_K + H_{RKKY} \]

\[
H_K = \frac{J_K}{N} \sum_j c^\dagger c S_{\beta\alpha}(j) \to -\frac{J_K}{N} \sum_{i,j} \left( (c^\dagger_{j\alpha} f_{j\alpha})(f^\dagger_{j\beta} c_{j\beta}) + \tilde{\alpha}\tilde{\beta}(c^\dagger_{j\alpha} f^\dagger_{j-\alpha})(f_{j-\beta} c_{j\beta}) \right)
\]

\[
H_M = \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(j) S_{\beta\alpha}(j) \to -\frac{J_H}{N} \sum_j \left[ (f^\dagger_{i\alpha} f_{j\alpha})(f^\dagger_{j\beta} f_{i\beta}) + \tilde{\alpha}\tilde{\beta}(f^\dagger_{i\alpha} f^\dagger_{j-\alpha})(f_{j-\beta} f_{i\beta}) \right]
\]

\[
H_K \to \sum_j \left[ c^\dagger_{j\alpha} \left( V_j f_{j\alpha} + \tilde{\alpha}\Delta^K_j f^\dagger_{j-\alpha} \right) + \text{H.c} \right] + N \left( \frac{|V_j|^2 + |\Delta^K_j|^2}{J_K} \right)
\]

\[
H_H \to \sum_{(i,j)} \left[ t_{ij} f^\dagger_{i\alpha} f_{j\alpha} + \Delta_{ij}\tilde{\alpha} f^\dagger_{i\alpha} f^\dagger_{j-\alpha} + \text{H.c} \right] + N \left[ \frac{|t_{ij}|^2 + |\Delta_{ij}|^2}{J_H} \right]
\]

Uniform solution:

\[
H = \sum_{k,\alpha>0} \left( \tilde{c}^\dagger_{k\alpha}, f^\dagger_{k\alpha} \right) \begin{pmatrix} \epsilon_k \tau_3 & V\tau_3 \\ V\tau_3 & \vec{w} \cdot \vec{\tau} + \Delta_{Hk}\tau_1 \end{pmatrix} \begin{pmatrix} \tilde{c}_{k\alpha} \\ f_{k\alpha} \end{pmatrix} + N_s N \left( \frac{|V|^2}{J_K} + 2\frac{\Delta_H^2}{J_H} \right)
\]
SP(N) Large N Approach.

\[ H = H_c + H_K + H_{RKKY} \]

Uniform solution:

\[
H = \sum_{\bf k, \alpha > 0} (\tilde{c}^\dagger_{\bf k\alpha}, \tilde{f}^\dagger_{\bf k\alpha}) \left( \begin{array}{cc} \epsilon_{\bf k} \tau_3 & V \tau_3 \\ V \tau_3 & \tilde{w} \cdot \bar{\sigma} + \Delta_{H\bf k} \tau_1 \end{array} \right) \left( \begin{array}{c} \tilde{c}_{\bf k\alpha} \\ \tilde{f}_{\bf k\alpha} \end{array} \right) + N_s N \left( \frac{|V|^2}{J_K} + 2 \frac{\Delta_H^2}{J_H} \right)
\]