Spin Liquids

What They Are NOT

Systems of Interacting Spins with
NO spin ordering
NO broken symmetries

No spin ordering

\[ q_{EA} = \lim_{t \to \infty} \lim_{N \to \infty} \langle S_i(t_0) S_i(t_0+t) \rangle \]

Average over long set of \( t_0 \)’s

= 0 \quad \text{ergodic}

\neq 0 \quad \text{system} \quad \text{"trapped" in single phase}
Absence of magnetic ordering of any type!

Not described by Landau theory

Brief discussion of Landau theory

Let's go back to an Ising FM

\[ M = \begin{cases} \uparrow\uparrow\uparrow\uparrow & \text{Disordered} \\ \uparrow\downarrow\downarrow\downarrow & \text{Order parameter} \\ \end{cases} \]

\[ M \neq 0 \quad T < T_c \]

\[ M = 0 \quad T > T_c \]
Landau theory = effective theory of an order parameter

Key assumptions

- at fixed value of the order parameter $M$, $f(M)$ is analytic ($f = \frac{E}{V}$) (non-analyticity at $T_c$ occurs because)
- in partition function must solve (over all values of $M$)

Ising model

$$Z = \sum e^{-\beta E}$$

($\sigma_i = \pm 1$)

Let $M = \sum_{j} \frac{\sigma_j}{N}$ in $N$ sites.
\[ Z = \sum_{M} \sum_{\sigma_f = \pm 1} \sum_{\sum_{\sigma_j} \sigma = MN} e^{-\beta E} \]

\[ e^{-\beta V f(M)} = \sum_{\sigma_f = \pm 1} \sum_{\sum_{\sigma_j} \sigma = MN} e^{-\beta E} \]

where \( V \) = volume

For large \( N, M \) is essentially continuous

\[ Z = \int_{-1}^{+1} dM e^{-\beta V f(M)} . \]
Landau's assumptions near $T_c$

- $f(M)$ analytic $\Rightarrow$ can be expressed as Taylor expansion near critical point
- $f(M)$ obeys symmetries of $H$

$$f(M) - f_0 = \frac{\lambda^2}{2} \left( T \right) M^2 + \frac{\beta}{4} M^4$$

$M \rightarrow -M \Rightarrow$ only even powers (no field) of $M$

$$\lambda(T) = \lambda \left( T - T_c \right)$$

$\beta > 0$ stability $\tilde{\beta} = \beta$
\[ \frac{df}{dM} = 0 \Rightarrow \alpha (T-T_c) + \beta M^3 = 0 \]

\[ M^2 = -\frac{\alpha (T-T_c)}{\beta} \]

\[ M(T) \propto |T-T_c|^{1/2} \]

\[ f-f_0 = \begin{cases} 
-\frac{\alpha^2}{2\beta} (T-T_c)^2 & T < T_c \\
0 & T > 0 
\end{cases} \]
Spin Liquids

- No Spin Ordering
- No Broken Symmetry
- No Landau description

Non-magnetic ground state
Built from well-formed local moments

Exotic Excitations

Exotic?

In most phases of matter, the excitations can be constructed from elementary excitations that are either electron-like \((s = \frac{1}{2}, q = \pm 2)\) or magnon-like \((\text{spin } s = 1, q = 0)\). Integer charge standard spin flip excitations are of the total spin...
Fractional?

e.g. Half-integer "spinons" connected by "tension-free" strings even at $T=0$ (emergent gauge theory takes care of global spin constraint)

Let's return to the 1D AFM (1d "spin liquid" due to Bethe) as setting to illustrate spinons.

Classical SLs very difficult to find:
  - Order by Disorder
  - Spin Freezing

Quantum Mechanics Needed!
Key aspect of Quantum Mechanics:

Superposition

Any linear combination of allowed quantum states

\[ \Downarrow \]

allowed state

(extended to many electrons)

Recall Néel vs Landau approach in AFM

\[ H = J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j \]

valence bond = entangled pair of spins
1973 Anderson: Eearth of 2D \( S = \frac{1}{2} \)

AFMs

Resonating Valence Bond

ground-state!

\( \text{(superposition of all valence)} \)
\( \text{(bond states)} \)

In valence bond spins are entangled.

Quantum entanglement of spins:

Quantum state of each spin in the group cannot be described independently of the state of the others in the group, even if they are separately by large distances.

Primary feature of Quantum Mechanics