1. Bishop 3.4

2. Bishop 3.7

3. **ML and Bayesian curve fitting**

Consider \( y(x) = a_0 + a_1 x^2 \) with \( a_0 = 1, a_1 = 2 \) and \( x \in [-3, 3] \).

Generate \( N = 100 \) datapoints by:

(i) randomly sampling \( x \) in the \([-3, 3]\) range using a uniform distribution

(ii) computing \( y(x) \)

(iii) computing \( t = y(x) + \varepsilon \), where \( \varepsilon \) is a random variable sampled from \( \mathcal{N}(\varepsilon | 0, 0.01) \) \( \sim \varepsilon^2 \) \( \text{[i.e. } \varepsilon = 0.1] \)

Consider a linear model of the form

\[ y(x, \underline{\omega}) = \omega_0 + \omega_1 x + \omega_2 x^2. \]

(a) Find the ML weights and plot \( y(x, \underline{\omega}_{ML}) \) alongside \( y(x) \)

[report \( \underline{\omega}_{ML} \) as well]
(b) Find $\beta_{\text{ML}}$ using $\hat{\beta}_{\text{ML}}$.

Use $\beta_{\text{ML}}$ and $d = 1.0$ to compute the predictive distribution in the $x \in [-3, 3]$ range. Plot the mean of the predictive distribution alongside with $\pm 6\sigma(x)$ curves [cf. Fig. 3.8] and $y(x)$, the "true" curve.

Draw samples from the posterior $\tilde{\omega}$ for $K = 10$ and plot the corresponding $y(x, \tilde{\omega})$ curves, alongside with $y(x)$ [cf. Fig. 3.9].