Physics 326: Computer Based Experimentation and Physics Computing

Instructor: Office: Phone: e-mail Office Hour: Prof. Weida Wu Serin W 117 848-445-8751 wdwu@physics.rutgers.edu By appointment

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AN INTRODUCTION TO Error Analysis THE STUDY OF UNCERTAINTIES IN PHYSICAL MEASUREMENTS SECOND EDITION John R. Taylor

Textbook: "An Introduction to Error Analysis", J.R. Taylor, 2nd Ed. University Science Books.

Web Site for Course:

http://www.physics.rutgers.edu/ugrad/326/



Lecture and Lab sections

Lecture(Wu):Wednesday (8:10-9:30PM)SEC 209Lab Sect. 3 (Zhang):Monday (6:40pm-9:30pm)Serin 101Lab Sect. 1 (Wu):Tuesday (10:20am-1:20pm)Serin 101Lab Sect. 2 (Zhang):Tuesday (3:20pm-6:20pm)Serin 101Lab Sect. 4 (Zhang):Thursday (6:40pm-9:30pm)Serin 101

No more than 2 people per group.

Finish data collection and/or analysis during lab session.

Preparation for the labs

Lab instructions are posted to the course web site. You are expected to download, print, and read these instructions before coming to lecture.

Each lab will be discussed in the Wednesday lectures before the lab.

In addition, you should understand what to do in the lab **BEFORE** coming to a lab.

Reports and quizzes

Lab Reports:

Lab reports are to be prepared individually and handed in during the lab session of the following week, i.e., you have one week to write your report. No late reports will be accepted. Copied lab reports will not be accepted. Do not write a report if you have not actually done the lab; it will not be accepted. Type and print your reports. No hand written report will be accepted.

Quizzes (5-7):

Short quizzes will be given occasionally during lectures through the semester. Topics in the quizzes are lecture and lab contents, reading assignments. Make-up quizzes will not be offered unless you have a documented medical reason for missing the quiz.

Grading

The course grade will be based mostly on the lab reports (\sim 90%), with the remainder determined by quiz scores and lecture attendance.

Grade cutoffs (Tentative)

Α	B+	В	C+	С	D	F
90	85	75	70	65	50	<50

Format of lab report

- Introduction (a short overview/background)
 - What is this about? Why is it interesting?
- Method
 - techniques, instruments, procedure, data analysis, error analysis
 - Do NOT copy from lab manual
- Results and discussion
 - tables and figures
 - connect the results back to the theory (intro)

• Conclusion

- one or two sentences
- **References** (if any)

Course schedule

Week	Lectures (Wednesdays)	Monday and Tuesday (sec. 1,2,3)	Thursday (6:40-9:30pm)		
Sept. 1 – Sept. 5 Labor Day week	Lec 1 (ppt, pdf) Sept. 3	No labs	1 - Propagation of Errors		
Sept. 8 – 12	Lec 2, (<u>ppt, pdf</u>) Sept. 10	1 - Propagation of Errors	2 - Wavelength of Light		
Sept. 15 - 19	Lec 3a, (ppt, pdf) Sept. 17	2 - Wavelength of Light	3a - Distribution Functions (I)		
Sept. 22 – 26	Lec 3b, (ppt, pdf) Sept. 24	3a - Distribution Functions (I)	3b - Distribution Functions (II)		
Sept. 29 - Oct. 3	Lec 4, (ppt, pdf) Oct. 1	3b - Distribution Functions (II)	4 - Least Squares Fit		
Oct. 6 – 10	Lec 5, (ppt, pdf) Oct. 8	4 - Least Squares Fit	5 - Damped Harmonic Motion		
Oct. 13 – 17	Lec 6, (ppt, pdf) Oct. 15	5 - Damped Harmonic Motion	6a - Forced Harmonic Motion (I)		
Oct. 20 – 24	No lecture	6a - Forced Harmonic Motion (I)	6b - Forced Harmonic Motion (II)		
Oct. 27 – 31	Lec 7, (ppt, pdf) Oct. 22	6b - Forced Harmonic Motion (II)	7a - Fourier Analysis (I)		
Nov. 3 – 7	No lecture	7a - Fourier Analysis (I)	7b - Fourier Analysis (II)		
Nov. 10 – 14	Lec 8, (ppt, pdf) Nov. 12	7b - Fourier Analysis (II)	8a - Onset of Chaos (I)		
Nov. 17 - 21	No lecture	8a - Onset of Chaos (I)	8b - Onset of Chaos (II)		
Nov. 24 – 28	Thanksgiving week	No labs	No lab		
Dec. 1 – 5	No lecture	8b - Onset of Chaos (II)	Make up labs		
Dec. 8 – 13	All lab reports due. Make up labs				

9 lectures, 12 labs, 8 reports, 5-7 quizzes

Lab 1: Propagation of error Errors, or uncertainties, are *inevitable* in measurements. Note that here "errors" mean **random** errors. One should always avoid **systematic** errors.



Errors: random vs. systematic



Statistical Analysis of random error

Random error is treated as a random variable that follow a random distribution.

Q: How to evaluate random errors?

A: Repeated measurements.



The mean and the standard deviation

The mean:
$$\overline{x} = \frac{x_1 + x_2 + \dots + x_N}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

N: # of measurements x_i : value of *i*th measurement

The standard deviation (error) of a single measurement:

$$\sigma_{x} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_{i} - \overline{x})^{2}} \quad \text{i.e.} \quad \delta x = \sigma_{x}}$$
$$X_{i} \pm \sigma_{x}$$

How about standard deviation of the mean?

The standard deviation of the mean

The mean:
$$\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

 $N: \text{ # of measurements}$
 $x_i: \text{ value of } i\text{th measurement}$

Standard deviation of the Mean (error of the average value):

$$\sigma_{\overline{x}} = \sigma_x / \sqrt{N}$$

$$\overline{x} \pm \sigma_{\overline{x}}$$

More discussion of this topic in lab 3.

How to report errors properly?

 $x \pm \delta x$

Measurement and error:

Rules of reporting error:

- 1. (measured value of *x*) = $x_{\text{best}} \pm \delta x$
- 2. Experimental uncertainties should almost always be rounded to one significant figure.
- 3. The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty.



An example

 $g = 9.82138 \text{ m/s}^2$, $\delta g = 0.02326 \text{ m/s}^2$

Rules of reporting error:

1. (measured value of *x*) = $x_{\text{best}} \pm \delta x$

 $g = 9.82138 \pm 0.02326$ m/s²



2. Experimental uncertainties should almost always be rounded to one significant figure.

$$\delta g = 0.023 \rightarrow \delta g = 0.02$$

3. The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty.

$$g=9.821 \rightarrow g=9.82$$

$$g = 9.82 \pm 0.02 \text{ m/s}^2$$

One exception of rule #2

If the leading digit in the uncertainty δx is a 1, then keeping two significant figures in δx is more reasonable.



$$g = 9.822 \pm 0.013 \text{ m/s}^2$$



The propagation of errors (one variable) If *q* is a function of one independent variable *x*,

$$q = q(x)$$
$$\delta q = \left| \frac{dq}{dx} \right| \delta x$$

Then

For example: q = ax $\delta q = |a| \delta x$ q = ax + b $\delta q = |a| \delta x$ $q = x^n$ $\frac{\delta q}{|q|} = |n| \frac{\delta x}{|x|}$ The propagation of errors (multivariable) Provisional rules: (for quick estimation)

$$q = q(x, y) \qquad \qquad \delta q \approx \left| \frac{\partial q}{\partial x} \right| \delta x + \left| \frac{\partial q}{\partial y} \right| \delta y$$

Examples:

$$q = x + y$$
, or $q = x - y$ $\delta q \approx \delta x + \delta y$

$$q = x \cdot y, \ q = x / y$$
 $\frac{\delta q}{|q|} \approx \frac{\delta x}{|x|} + \frac{\delta y}{|y|}$

More precisely:
$$\delta q \leq \left| \frac{\partial q}{\partial x} \right| \delta x + \left| \frac{\partial q}{\partial y} \right| \delta y$$

Propagation of independent errors

If the uncertainties δx , δy are independent of each other,



This can be generalized to multivariable functions:

$$\delta q(x_1, x_2, \cdots, x_N) = \sqrt{\left(\frac{\partial q}{\partial x_1} \delta x_1\right)^2 + \left(\frac{\partial q}{\partial x_2} \delta x_2\right)^2 + \cdots + \left(\frac{\partial q}{\partial x_N} \delta x_N\right)^2}$$

Lab 1: Torsion Pendulum



M: Modulus of rigidity

Torsion Pendulum $k = \frac{4\pi^{2}}{T^{2}}I = \frac{4\pi^{2}}{T^{2}} \cdot \frac{m}{12}(a^{2} + b^{2}) = \frac{\pi^{2}m}{3T^{2}}(a^{2} + b^{2})$ $M = \frac{32L}{\pi d^{4}}k = \frac{32L}{\pi d^{4}} \cdot \frac{\pi^{2}m}{3T^{2}}(a^{2} + b^{2})$

Modulus of rigidity: (a.k.a. modulus of torsion, the shear modulus of elasticity)

$$M = \frac{32\pi}{3} \frac{Lm\left(a^2 + b^2\right)}{d^4 T^2}$$

Quantities to measure

- *L*: length of the steel wire
- d: diameter of the steel wire
- *m*: mass of the rectangular block
- *a*: width of the rectangular block
- *b*: length of the rectangular block
- T: period of torsional oscillation