Intermediate Quantum Mechanics
Notes for Lecture 1 (1/21/15)

Structure of Physics, Classical Physics and Quantum Mechanics vs. Classical Physics

The Structure of Physics

- The two pillars of physics are the Special Theory of Relativity (SR) and Quantum Mechanics (QM). All the rest of physics is supported by these two pillars. Any experimental disagreement with with either SR or QM would be the collapse of all of physics. This might happen but is extremely unlikely. Any experiment showing disagreement with either theory should be considered highly suspect. These theories are very well established and no one is doing serious research to test them. It would almost certainly be a waste of time. The active areas of research in physics are in those areas supported by these two pillars: atomic physics, nuclear physics, solid state physics, astrophysics, elementary particle physics, cosmology, etc. In these areas we don't understand everything and the forefront of knowledge is advancing.

- Classical physics (essentially Newtonian Mechanics) is a very good approximation at the scale of our everyday lives. It is very intuitive to us and satisfies our "common sense". That's because a working knowledge of classical physics has great survival value and so our brains evolved to "understand" classical physics. Quantum Mechanics and Special Relativity don't make common sense. The regimes where their effects are manifest are not encountered in our daily lives and therefore there is no survival value to having a working knowledge of them. (At least not until modern times, of course). The only things that matter in determining whether a theory is correct are:

  - Do it agree with all experiments (measurements)?
  - Is it mathematically consistent?

  Whether or not it makes common sense to us is irrelevant. In fact, relativity and quantum mechanics almost always don't. A serious mistake that students new to these theories make is to think that their calculation is wrong because it doesn't "make sense".

- Special Relativity was put forward by Albert Einstein in a set of two papers in 1905. It is important in the regime where velocities are close to the speed of light.

  $$v \sim c = 3.0 \times 10^8 \text{ m/s}$$

  At that speed you could go around the world about 7.5 times in one second.

- Quantum Mechanics was developed slowly in fits and starts over a period of about thirty years from 1900 to 1929. It was the work of many great physicists: Einstein, Planck, Bohr, Heisenberg, Schrodinger, Pauli, Born, Dirac, etc. It is important in the regime where distance is comparable to Planck's constant over the momentum.

$$x \sim \frac{h}{p}$$

Planck's constant, h, is the fundamental constant of QM. We more often use $\hbar = h/2\pi$. As we will see, the energy of particle is related to its frequency. We can write this as either $E = h\nu$ or $E = \hbar\omega$. They are of course equivalent. We will often have expressions with oscillatory functions. It's much more convenient to write, for example, $\cos(\omega t)$ than $\cos(2\pi\nu t)$. That's really the only reason for choosing $\hbar$ over h.

- The value of $\hbar$ is 1.05×10^{-34} J·s. It's good to see this number to appreciate how small it is when using macroscopic (SI) units. But, when we deal with quantum mechanics we are studying particle with very small masses and energies. It make no sense to calculate in term of Joules. Almost everybody can remember the value of c. The way to remember the value of $\hbar$ is by the product of $\hbar$ and c.

  $$\hbar c \;=\; 200 \text{ eV}\cdot\text{nm} \;=\; 200 \text{ MeV}\cdot\text{fm}$$

  [The more precise number is 197 but 200 is good enough. An electron volt (eV) is the kinetic energy gained by an electron when it falls through a voltage difference of 1 V (1 eV $= 1.6 \times 10^{-19}$J). One Fermi (fm) $= 10^{-15}$ m is about the size of the proton.]

- The program of physics is very simple to state and is the same for both classical physics and QM. It involves the following.
  - Select the closed system that you are studying. This could be a single particle or several particles with various types of interactions.
  - Determine what are the possible states of the system and how they are specified.
  - Determine how the system evolves (moves from state to state) in time. This is the dynamics.

Classical Physics

- In classical physics, the dynamics is given by Newton's Law:

  $$-\frac{dU(x)}{dx} \;=\; F(x) \;=\; ma$$

  If we know the position of the particle, then we know the force acting on it and therefore the rate of change of the velocity.

  $$dv \;=\; a\,dt \;=\; \frac{F}{m}\,dt$$

  This is not enough to know how the particle will evolve. We have to be given the velocity as well as the position at some particular time. The state of the system, in this case a single particle in one-dimension, is given by specifying its position and velocity. Then we have:

  $$dx \;=\; v\,dt \qquad\qquad dv \;=\; \frac{F}{m}\,dt$$

If we know the position and velocity at a given time we can calculate the position and time at an infinitesimal time dt later. We can then take the new position and velocity to calculate them at the next later time and so on.

- The state of a single particle in one dimension is given by specifying its position and velocity. Since momentum, $p = mv$, is a more useful concept than velocity, particularly so in quantum mechanics, we specify the state by giving its position and momentum. All the possible states of the system are represented by the set of all points in phase space, the (x, p) plane. For a particle in three dimensions, the phase space is 6 dimensional and for a system of N particles in three dimensions it is $6N$ dimensional. Since the changes in position and momentum during an infinitesimal time are infinitesimal, the evolution (motion) of the system is represented by a continuous trajectory in phase space .

- Classical physics is deterministic. If we know the state of the system at some time, then the entire future of the system can be determined with certainty. The entire past of the system can be determined with certainly as well. We just take dt in the above expressions to be negative. In practice, this complete determinism can't be achieved. There are two reasons for this. Because of the imperfection of any measuring device, we can't determine the state of the system with certainty, but this is a rather trivial constraint. We can always make our measurement more precise and in the ideal limit of perfect precision we will know the definite state of the system. The other reason is more significant. Often there are so many particles in the system that it isn't possible to know the position and momentum of each. There are, for example, about 10^{28} molecules in the air in the classroom. There is no way to know the exact state of the air. Instead we use probability and statistics to account for our lack of information (ignorance). The probability distributions of the position and the momentum of the molecules in the room can be known. We then speak of averages and other moments of the probability distributions. For example, if we sample many molecules in the room, we know what the average momentum (or velocity) of these molecules will be with extremely high precision. This use of probability is not fundamental. Classically, the air in the room really is in a definite state. We just don't know what it is.

Quantum Mechanics

- The situation with quantum mechanics is very different.

- In QM, the state of a system is not a set of points in phase space but rather it is an element of a complex vector space. This is the most basic postulate of quantum mechanics and accounts for most of its "weirdness". It means that quantum mechanics does not follow the logic of set theory but instead follows the logic of vector spaces.

- In QM, the position and momentum of a particle cannot be definitely specified simultaneously. They are "incompatible" observables. As a result, there is no sense to a trajectory in phase space. In fact, at the basic level of quantum mechanics, there isn't a concept of velocity. As we will see, the state of a particle is specified by specifying its wavefuntion, a function of position. This function of position also contains all of the information about the particle's momentum.

The distribution in position and momentum are not independent. We'll see what all of this means in a few weeks.

- As in classical physics, we might not know the exact state of a system either because the person who prepared the system didn't tell us what state they prepared it in or because there are so many particles in the system that, as in the classical case, we can't practically know all of the information. In these situations, we use probability just as in classical physics to account for our lack of information.
- In QM there is, in addition, a different and fundamental role of probability that doesn't exist in classical physics. In QM, even if we have complete information, that is, we know exactly the state of the system, we cannot make a definite prediction of what will be the outcome of a measurement (experiment). It's not because there is some inner workings of the system that we don't know about. Even if we know the exact state the outcome of a measurement will be random. This is the part of quantum mechanics that bothered Einstein and many others. "God doesn't play dice." But, remember, there is no requirement that QM agree with our common sense (even Einstein's common sense) the world is what it is. An example of the fundamental nature of probability in quantum mechanics is the decay of a muon. A muon is a heavy electron that decays with an average lifetime of 2.2 μs. This is the average decay time. If we prepare many muons some will decay earlier and others will decay later. The distribution of decay time will follow an exponential, $e^{-t/\tau}$ where $\tau = 2.2$ μs. We cannot tell when an individual muon will decay even though all muons are exactly identical. We can only give the probability of when it will decay.
- There is another basic difference between quantum mechanics and classical physics but here it is quantum mechanics that makes sense and classical physics that is clearly wrong. In classical physics, we assume that we can make a measurement on as system, for example, measure the position of a particle, without disturbing the system. If you think about it this is clearly nonsensical. If we want to know where a ball is we need to scatter light off of it with the scattered light going into our eyes or into a detector. The light carries momentum and energy and will disturb the state of the ball when it scatters off of it. Of course, because the ball is very massive, the effect of the scatted light can be ignored but it is still there. At the level at which quantum mechanics becomes manifest, the affect of the measurement of the system cannot be ignored. For example measuring the position of an electron will have a dramatic effect on tis state. In quantum mechanics, the affect of the measurement is an integral part of the theory. A measurement always affects the system being measured but macroscopically (in the realm of classical physics) this can be ignored.

Summary

- In classical physics:
  - The state of a system is a point in phase space.
  - The system follows a continuous trajectory in phase space.
  - The future and past of the system is completely determined.

  - We use probability to account for our lack of information about the state of a system.

- In quantum mechanics:
  - The state of a system is an element of a complex vector space.
  - The position and momentum of a particle are not definite. The system does not follow a trajectory.
  - A description in terms of probability is fundamental. Even with complete information we cannot make definite predictions.
  - Measuring a system affects the state of the system.