Advanced General Physics I
Fall 2019
Lecture 1
Classical Mechanics

In this course, we will cover classical mechanics. Classical mechanics was developed by Isaac Newton in the 17th century and it was all of physics for over two hundred years until near the end of the 19th century. Over those two hundred years, many great physicists (Lagrange, Hamilton, Poisson and others) developed sophisticated formulations of classical mechanics that allowed solutions of complicated problems involving multiple particle systems subject to constraints. These were, however, purely mathematical developments. The physics of classical mechanics was set forward in its entirety by Newton.

Classical mechanics is the physics of how the state of a system consisting of one or more point (Newtonian) particles evolves as a function of time. That is, given the state of the system at an instant of time it allows us to calculate what the state of the system will be in the next (or previous) instant. For example, if we treat the simple case of one particle in one spatial dimension, its motion is described by plotting its position as a function of time. Classical mechanics allows us to determine this curve for all times in both the future and the past.

Reasons for Studying Classical Mechanics

Classical mechanics describes physics very well at the scale of our everyday lives. At this macroscopic scale, it is an excellent approximation of the deeper underlying theory of quantum mechanics, but fundamentally it is completely wrong. Since it isn't a correct theory, it might seem strange that we would spend time studying it. Why don't we instead jump right to the correct theory of quantum mechanics? There are several reasons for this and everyone including all great physicists start by first learning classical mechanics. Here are some of the more important reasons.

1) Classical mechanics has important historical significance. While, in general, when studying physics it isn't very helpful to be concerned with its historical development, it is valuable to know about the accomplishments of the great physicists of the 18th and 19th since we can gain valuable insights by studying and becoming familiar with their work.

2) Classical mechanics is intuitive. Because the physics that we experience in our everyday lives had important survival value, the brains of humans and other animals evolved an intuitive understanding of physics as described by classical mechanics. We intuitively know what will happen when, for example, a ball is thrown in the air. Its behavior doesn't seem strange to us. Even a dog or a young child are able to catch a thrown ball. They are in a sense solving for the motion of the ball although certainly not in a mathematical form. If our ancestors had not known how a spear would fly through the air or how a tiger would jump, they would not have faired very well. Knowledge of quantum mechanics, on the other hand, had no survival value and so our brains didn't evolve to have an intuitive sense of it. The quantum mechanical behavior of systems at the microscopic level does not at all follow our intuition or common sense. It is, therefore, best to start our study of physics with classical mechanics, a theory that "makes sense" to us.

3) There are three important dynamical quantities in physics: energy, momentum and angular momentum. These arise from the basic principles of quantum mechanics but, if we

first encountered them there, they would be abstract mathematical quantities. However, at the macroscopic scale, they become the classical quantities of energy, momentum and angular momentum. From experiences in our everyday lives, we have a concrete understanding of what these are. We know that a moving ball has energy and when it hits us we realize it has momentum. We also intuitively understand the angular momentum of a spinning top or a spinning ice skater. If we first learn about these concepts in classical mechanics, we are then more readily able to relate to the more abstract concepts of energy, momentum and angular momentum in quantum mechanics.

4) Although it isn't a correct theory, classical mechanics is an extremely good **effective theory** at the macroscopic scale. Many of the problems that we might want to solve at this scale are best handled by classical mechanics. If we want to describe the flight of a ball, the motion of a roller coaster, the orbit of a planet or the trajectory of a spacecraft, we would best use classical mechanics. Trying to solve problems at this scale using quantum mechanics would be silly.

5) In developing mathematical formulations of the physics first proposed by Newton, physicists of the 18th and 19th century discovered concepts that were far deeper than classical mechanics itself. We will soon see examples of these. They include the principle of stationary action, Lagrangians, Hamiltonians and Poisson brackets. The early physicists were getting a glimpse of the deeper level of quantum mechanics on which classical mechanics is based, but they had no realization that was what they were seeing. That would have to wait until the heroic efforts by physicists in the early 20th century. In terms of our study of fundamental physics, this is the most important reason for starting with classical mechanics. Here in the more intuitive realm we will get our first exposure to these deep principles that will then carry through all of physics.

Fundamental Principles of Physics

Before we begin our study of classical mechanics, we will first discuss important basic, universal principles of physics. They apply to all areas of physics: quantum mechanics, special relativity, general relativity, etc. They are the basic postulates (assumptions) on which all of physics is built. These are the deepest principles of physics extending well beyond classical mechanics. They greatly constrain what the possible laws of physics can be. These basic principles are:

- Information is conserved.
- The laws of physics are the same everywhere.
- The laws of physics are the same in all directions.
- The laws of physics are the same in all reference frames moving at uniform (constant) velocity.
- The laws of physics don't change with time.
- The equations of motion that describe the time evolution of a system are given by the Principle of Stationary Action.

We'll discuss each of these in a bit more detail.

Information is conserved

Information is never created or destroyed. This might seem strange to you since, if you are like me, you lose information all the time. However, information isn't really lost; it just becomes inaccessible. For example, you might think that if you burn a book the information contained in the book is lost, but that isn't correct. The information that was contained in the state of the ink on the pages of the book is now contained in the state of all of the air molecules and all of the ashes that resulted from burning the book. In principle, you could recover this information. Of course, doing this isn't practical and so the information has been effectively lost. We describe this as an increase in entropy which quantifies our ignorance or lack of information, but the information content of the universe hasn't changed.

A very important result of this principle is that the laws of physics must be reversible. That is, if the direction of time is reversed, the laws of physics must be the same. If you know the state of a closed (isolated) system, then you have all the information needed to determine the entire past and entire future of the system.

Invariance of the Laws of Physics on the Frame of Reference

The laws of physics are invariant (unchanged) if a closed system is either translated (moved) to a different point in space, rotated in space or moved at a constant velocity (constant speed with no change in direction). This is summarized by saying that the laws of physics are the same in all inertial reference frames where an inertial reference frame is the frame of an observer that is moving at a constant velocity. You might think that this isn't true. For example, if I'm standing up straight and drop a ball it falls toward my feet. So, I could make a law of physics that an object that I drop falls to my feet. But now if I lean over against a wall and drop the ball, it doesn't fall toward my feet so my law of physics is not invariant under rotation (leaning). But that is because I have a wrong law of physics. The correct law is that the ball accelerates in the direction of the gravitational force. When I'm standing up straight the gravitational force is toward my feet but when I'm leaning over it's not. This law of physics, that the ball accelerates in the direction of the force, is invariant under rotations. Similarly, if I drop a ball on the Moon, it falls with a smaller acceleration than on the Earth. That's not because the law of physics is different on the Moon than on the Earth. It's because the environment (the strength of the gravitational force) is different on the Moon than on the Earth. The laws of physics are the same on both the Moon and the Earth, they are just being applied in different environments.

The invariance of the laws of physics allows us to distinguish between correct and incorrect laws. Correct laws are those that are invariant. But how do we know that this postulate is true and in fact is it actually true? The universe originated from a Big Bang that occurred 13.7 billion years ago. Since no information travels faster than the speed of light, we can only know about the universe that is less than 13.7 billion light years away (actually about 45 billion light years away since the universe has expanded by about a factor three since the light was emitted 13.7 billion year ago). That is a very large distance, about 10^{27} m, but we don't know how big the universe actually is. It is at least a hundred times larger than the universe that we can see (the visible universe) but it could be much larger. It could even be infinite in size. We don't know what the laws of physics are in parts of the universe that we can't see and it's possible that they are different. We do know that within the visible universe to as accurately as we've been able to measure, the laws of the physics are the same everywhere. We assume (postulate) that they are exactly the same everywhere.

Invariance of the Laws of Physics with time

As far as we have been able to determine, the laws of physics also seem to be independent of the moment in time. Of course, the conditions of the universe have changed with time. The early universe was much hotter and denser than it is today. What took place in the early hot and dense universe was different than today but it was governed by the same laws of physics. The laws are just applied in different situations. Just as today, what takes place in the interior of the Sun is very different from what takes place on the Earth but both cases are governed by the same laws of physics, again just applied to different situations.

Principle of Stationary Action

The **Principle of Stationary Action** is fundamental to all theories of physics including Classical Mechanics, Classical Field Theory, Quantum Mechanics, Quantum Field Theory and Special and General Relativity. All laws of motion are derived from it. The principle states that the motion (trajectory) of a particle between two points in space is such that the true trajectory of the particle is the one for which the action is stationary. At the moment, you likely don't know what this statement means. That's OK, this is just a preview. In a couple of weeks, will see how to calculate the action, what stationary action means and how it leads to the equations of motion.

Conservation Laws

The invariance of the laws of physics directly lead to conservation laws for closed systems. There are three dynamical quantities that are conserved for closed systems: momentum, angular momentum and energy. Because these quantities are conserved they play important roles in physics. Invariance under spatial translations results in conservation of momentum, invariance under rotations results in conservation of angular momentum and invariance under time translations results in conservation of energy. We will later see in detail how these relations between invariance and conservation laws come about.

Homogeneity and Isotropy of Space and Time

In addition to the invariance of the laws of physics, it is often also assumed that the universe itself is homogeneous (the same at all points) and isotropic (the same in all directions). Note that this is not the same as saying that the laws of physics are the same everywhere and in all directions. The laws of physics are invariant but the universe (the environment) might be different from point to point and in different directions. On a local scale, the universe is clearly neither homogenous nor isotropic. The room I'm sitting in has chairs and a desk at some locations and windows at others. Also it is not the same in all directions. In one direction I can walk through the door while if I walk in another direction I'll bump into a wall. If the assumptions of homogeneity and isotopy are true, they are only so when averaged over a scale of billions of light years: larger than my room, the Earth, the Solar System, the Galaxy and galactic clusters. Homogeneity and isotropy are usually assumed to be true on the scale of billions of light years when we are studying the large scale structure of the universe. The universe might be homogenous on the scale of billions of light years but the assumption that the universe is isotropic is definitely not precisely true. We know from precision measurements that the Cosmic Microwave Background Radiation, a remnant microwave radiation left over from the early age of the universe is not isotropic but varies from direction to direction by about one part in a hundred thousand (10^{-5}).

The Framework of Physics

There is a simple framework or structure of physics that applies in all cases. If you keep this simple structure in mind, it can help you to avoid becoming confused by nonessential elements when thinking about the physics. The framework consists of: **the system** that is under study, **the stage** on which the system moves, **the states of the system**, **the environment** acting on the system and the **equations of motion** that prescribe the evolution of the system from state to state as a function of time.

1) **The system.** The system consists of the object or objects that are under study.

2) **The stage.** This is the platform on which the process takes place.

3) **The states of the system.** The states of a system are specified by giving all the information that is both necessary and sufficient to completely describe the system.

4) **The environment.** The environment consist of everything that acts on the system and that is not part of the system itself. If there are no environmental factors acting on the system, then it is said to be a **closed system**.

5) **The equations of motion.** The equations of motion describe the evolution of the system from one state to another as a function of time. They are often called "the physics".

The Framework of Classical Mechanics

We'll next describe the framework for the theory of classical mechanics.

The system

The system in classical mechanics consists of one or more point particles, sometimes referred to as Newtonian particles. These are taken to be mathematical points occupying no volume. Newton realized that the regularities of nature meant that there must be common underlying building blocks. He, of course, didn't have any knowledge of the structure of atoms or of subatomic particles and he certainly didn't have a theory that could explain atoms or solid matter but he understood that if a particle were to be a fundamental building block it must be a point with no size or structure since, if it were to have a structure, it would need to be made up of more fundamental objects to account for its size. For example, atoms are made up of electrons and atomic nuclei, nuclei are made up of proton and neutrons and protons and neutrons are make up of quarks.

These point-like Newtonian particles have certain intrinsic properties. One intrinsic property is the particle's **mass**. In classical mechanics, there is no explanation of mass nor why a particle's mass has a certain value. It is just taken as given. The one other intrinsic property of a classical particle is its **electric charge**. The mass of a particle is always positive while the electric charge can be positive, negative or zero. For now, we will only be concerned with particle mass. We won't need to consider electric charge until next semester when we discuss the interaction of particles with electric fields.

In this course, we will often consider problems involving composite objects such as: a ball, a block, a car, etc. We are to think of these as made of up of a large number of Newtonian point particles that are somehow rigidly attached so that they don't move with respect to each other but all move together as whole. The mass of the composite object is equal to the sum of the masses of its constituents, $M = \sum_i m_i$, where m_i is the mass of constituent particle i. Since the composite object is taken to be rigid, it has no internal motions and so we can treat the entire object as a particle with mass M.

The stage

For most of this semester we will study non-relativistic mechanics. This applies for speeds much less than the speed of light, $c = 3.0 \times 10^8$ m/s (186,000 miles/s). The stage then consists of the separate entities of space and time. That is, objects are described as moving through space as a function of time. Next semester, we will study relativistic mechanics in which objects move at speeds up to the speed of light. We'll see then that space and time aren't separate but are actually mixed together to form **spacetime**. For this semester, we will consider space and time to be totally separate entities.

The states of the system

In classical mechanics the state of a system is given by specifying the positions and the velocities (or equivalently the momenta) of all the particles in the system. For example, if the system consists of just a single particle and we consider only one spatial dimension, then the state of the system (particle) is given by specifying its position (x) and it velocity (v).

The environment

The **environment** represents the external forces that act on the particles in the system. Examples of external forces are: a system acted on by a gravitation force due to an external object, an object attached to the end of a spring subject to a force due to the external spring, an object pushed on by an external rod, surface or hand, etc.

A **closed system**is one with environmental influences, that is, no external forces acting on it. As we will see, closed systems are of particular interest since their energy, momentum and angular momentum are conserved. For a non-closed system that is subject to external forces, if we include the objects that are producing the external forces as part of the system then this expanded system is closed with no external or environmental forces. For example, a system subject to a gravitational force becomes a closed system if the object producing the gravitational force is included in the system. The universe itself s itself a closed system since, by definition, the universe includes everything there is.

The equations of motion

In classical mechanics, the **equation of motion** that describe how the system evolves as a function of time is given by the famous equation of Newton, $F = ma$. That is, a force acting on a particle causes it to accelerate in the direction of the force with a value of acceleration inversely proportional to the particle's mass. If the force may be due to other particles in the system, it is called an internal force while if it is due to the environment it is called an external force.