Identification of Variables in Model Tracing Tutors

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1 Introduction

Model tracing tutors (MTT) monitoring a student's problem solving efforts benefit greatly when all decisions made by the student are explicitly represented. Unfortunately, even simple problem solving usually involves too many decisions to make this practical. For example, students solving an introductory university-level physics problem need to use numerous variables and instantiate several relevant laws of physics. When a student is required to explicitly provide all details about his decisions the task becomes unwieldy and the student looses interest and motivation. One model tracing tutor[2] requiring complete explicit specifications takes an average of 22 minutes for a student to do a single problem.

Beginning students are often vague and careless in their definitions of variables and it is useful for a tutor to require explicit information from them. As the student becomes more proficient, he internalizes much of this explicit process, and tutors should respond by relaxing the requirement for explicit specifications (i.e., scaffolding) where possible[3]. This presents a challenge to the MTT - how much of the scaffolding can be removed without reducing the MTT's ability to monitor a student's progress.

This paper describes an approach for automatically identifying the type of a variable (i.e., mass, energy, momentum) and for generating the appropriate feedback when the student uses a variable inconsistently. Clues to identifying the variables come from the variable names chosen and from the equations presented. For example, in a problem involving masses, accelerations, tensions, and movement a variable beginning with T usually represents a time or a tension and not an acceleration. Our approach associates with each variable used by the student a set of possible physics concepts, each of which carries a definite combination of physical dimensions. Thus T would have associated with it two possible physics concepts: time, with dimensions of seconds (s), and tension, with dimension of force, ($\mathrm{kg} \cdot \mathrm{m}/\mathrm{s}^2$).

Each equation, in fact each operator within each equation imposes constraints on the possible dimensions of the terms. This paper investigates how effectively these heuristic naming rules and dimensionality constraints can determine the meaning of student-defined variables, and also identify the source of inconsistency when students make mistakes.

2 Approach

A constraint graph is built where variables in each equation are instantiated as leaf nodes and internal nodes represent operators [1], e.g., $+, -, *, /, =$, and functions, e.g., $\cos, \sin, \tan$. Each node's value is the set of possible dimensions for that node. The possible dimensions of leaf nodes are obtained from a knowledge base matching name strings to physics variables and their associated dimensions. Most of these strings are prefixes; any variable name beginning with the string has the associated concept's dimensionality added to its list of possibilities. Some prefixes preempt others, preventing variables such as `theta1` from being considered as a time variable, even though it begins with the letter t. Some other strings, such as `G` for Newton's gravitational constant, match only if they are the whole string and not a prefix.

Constraint propagation is used to analyze dimension information among the nodes and to narrow the possibilities as well as identify inconsistencies. To give meaningful feedback when a student's mistake leads to inconsistencies, we found that it was best to *localize* information propagation as much as possible. This differs from standard constraint propagation algorithms that focus on efficient representation and propagation of information by eliminating choices as quickly as possible. Our heuristics focus instead on finding the smallest inconsistent subexpression. In this domain it is more useful to localize a mistake than to quickly determine whether or not the entire submission is consistent or not. To support this need for locality we treat each occurrence of a variable independently rather than build in the assertion that they represents a single concept. We augment our representation with **identity** constraints between identical variables and apply these constraints only after algebraic ones have completed.

Information is propagated and checked in several sequential phases: (1) the dimensions of the variables (leaf nodes) are propagated upwards, (2) information is propagated downwards within an equation, (3) identity constraints within an equation are validated, and (4) identity constraints between equations are validated. The effect is to maximize the inference of dimension values locally before propagation to other nodes.

3 Experimental Evaluation

The approach was first evaluated on roughly 350 answers to four physics problems from 88 different students in an introductory physics course for engineers and science majors. Only 5% of the submitted answers (two to three answers for each problem) were ambiguous and required additional information from the student. The technique was subsequently evaluated on equation sets extracted from the log files of the ANDES system [4] from fall, 2000. ANDES is also a tutoring system for introductory college level physics. It has a large database of problem types and is in current use at the United States Naval Academy.

The ANDES system permitted the students to use numeric values, with or without units, in place of variables, e.g., 9.8 instead of g for gravity. Consequently, constants can sometimes have unstated dimensions and our system has to treat each constant initially as having all dimension possibilities. In the evaluation, we found that there were many equation sets where the dimensions of all the variables were determined but the dimensions of some of the constants were not. Our results are shown in Table 1.

In 83% of the cases the dimensionality of every variable and every constant was uniquely determined. Considering only variables and ignoring constants, we found that in 89% of the equation sets the dimensionality of all variables were determined. In 3% of the cases we found that exactly one variable was ambiguous so that with at most one clarifying question to the student we could uniquely determined the dimension of all variables in 92% of the cases.

Of the remaining 8% of the cases, 6% had more than one ambiguous variable and 2% (198) were found to be dimensionally inconsistent. The knowledge base that we used had 109 entries and contained information covering all of Newtonian mechanics, the area from which the analyzed corpus was obtained.

No ambiguous variables	8761	89%
One Ambiguous variable	267	3%
Two or more Ambiguous variables	639	6%
Inconsistent Dimensions	198	2%

Table 1: Evaluation on the ANDES data. A partitioning of the equation sets by the number of variables whose dimensionality could not be uniquely determined.

The results show that without any special information about ANDES, e.g., variable naming conventions, our technique can determine the dimensions of all the variables in roughly 90% of the sets of equations, even when most of the sets are incomplete. In cases where ambiguity remains, asking the student to clarify the meaning of a few variables may be enough to determine all dimensions. The experiments also showed that our system using identity constraints was able to correctly identify the source of inconsistencies, something it was unable to do when identical names were treated as one variable from the beginning.

4 Conclusion

This paper has shown how domain knowledge combined with heuristic constraint propagation is used to determine the context and implicit information contained in student answers, specifically the dimensions of variables in systems of equations. This approach has been tested and evaluated on answers from students at two colleges. The results show that the technique uniquely determined the dimensions of all the variables in 89% of the sets of equations. By asking for dimension information about one variable, the percentage of sets in which all variables have unique dimensions increases to 92%.

References

[1] C. W. Liew and D. E. Smith, Checking for dimensional correctness in physics equations. In *Proceedings of Fourteenth International Florida AI Research Society Conference* (2002).

[2] Shelby, R. N.; Schulze, K. G.; Treacy, D. J; Wintersgill, M. C.; VanLehn, K.; Weinstein, A., *An assessment of the Andes tutor*, Proceeding of the 5th National Physics Education Research Conference, July 21-25, 2001, Rochester, NY.

[3] VanLehn, Kurt, Freedman, Reva, Jordan, Pamela, Murray, Charles, Osan, Remus; Ringenberg, Michael; Rosé, Carolyn, Schulze, Kay, Shelby, Robert; Treacy, Donald, Weinstein, Anders, and Wintersgill, Mary, *"Fading and Deepening: The Next Steps for Andes and other Model-Tracing Tutors"*, Proceedings 5th International Conference, ITS 2000,Montreal Canada. p474-

[4] A. S. Gertner, Providing feedback to equation entries in an intelligent tutoring system for physics. In *Proceedings of the 4th International Conference on Intelligent Tutoring Systems* (1998).