Physics 618 Twenty-fifth Lecture April 21, 2017

On Tuesday we saw that starting with a local symmetry rather than a global one makes a dramatic change in the effects of dynamical symmetry breaking. The gauge symmetry, which tells us that some of the apparent degrees of freedom in $\mathcal{A}_\mu$ and ϕ_k do not correspond to physical ones, but are gauge artifacts which should be annihilated by imposing gauge conditions, instead of killing the longitudinal modes of the gauge particles annihilate the Goldstone bosons. We saw several examples, but the most important, of course, is the Standard Model of Electro-Weak interactions, based on the local symmetry SU(2)× U(1) with a doublet Higgs boson, where three of the real components of the Higgs are eaten by the $W^\pm$ and Z gauge particles to become massive, while the fourth gauge particle remains massless and is the electromagnetic field. The parameters of the theory, the mass of the Higgs and the couplings (charges) of it to the two gauge groups are reconfigured into the electric charge $e = g\sin\theta_W$, the Weinberg (or weak) angle θ_W, and the Higgs mass, which are now all measured experimentally.

But the theory we considered on Tuesday had no fermions, no quarks and no leptons, so we have much to add today.

Today

Adding in the leptons is a bit tricky because the weak interactions do not respect parity invariance. The spinor representation of the isochronous but improper Lorentz group, that is, one invariant under Lorentz transformations which preserve the forward nature of time but not necessarily of right-handedness, is a four dimensional complex representation called a Dirac spinor. The Lorentz symmetries are products of four-dimensional γ matrices. But the weak interactions are not parity invariant, so the relevant symmetry group is the *proper* isochronous Lorentz group, and the Dirac spinor is not irreducible but falls apart into two two-complex or four-real dimensional representations, left and right handed respectively. Thus our SU(2)×U(1) gauge group will act differently on the left-handed and right-handed components of the lepton or quark spinors.

The weak interactions were first discovered in beta decay, in which a proton turns into a neutron or vice-versa, releasing a positron or electron, as well as a neutrino. As the leptons (electron and neutrino) are conserved, we must view this as a charged gauge particle coupling to the nucleon (or constituent quark) on the one and and the leptons on the other. So the parts of these spinors which couple to the SU(2) are doublets differing in electric charge by one unit.

Fermi found beta decay could be described by the four-fermion interaction, which also describes mu-meson decay:

$$\mathcal{M} = \frac{g^2}{2M_W^2}\bar{\psi}_e\gamma^\rho(1-\gamma_5)\psi_{\nu_e}\bar{\psi}_{\nu_\mu}\gamma_\rho(1-\gamma_5)\psi_\mu$$

which is the limit, with $M_W \to \infty$ of a μ^- giving off a W^- to become a ν_μ neutrino, and the W^- then decaying into an electron and an anti-electron neutrino. But the form of the coupling, $\bar\psi_{\nu_\mu}\gamma_\rho(1-\gamma_5)\psi_\mu$ shows that the W only couples to the left-handed part of the leptons.

	$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L$	$\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L$	$\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L$	$\begin{pmatrix}u\\ d'\end{pmatrix}_L$	$\begin{pmatrix}c\\ s'\end{pmatrix}_L$	$\begin{pmatrix}t\\ b'\end{pmatrix}_L$
Q	$\begin{pmatrix}0\\ -1\end{pmatrix}$	$\begin{pmatrix}0\\ -1\end{pmatrix}$	$\begin{pmatrix}0\\ -1\end{pmatrix}$	$\begin{pmatrix}2/3\\ -1/3\end{pmatrix}$	$\begin{pmatrix}2/3\\ -1/3\end{pmatrix}$	$\begin{pmatrix}2/3\\ -1/3\end{pmatrix}$
y	-1	-1	-1	$1/3$	$1/3$	$1/3$

	e_R	μ_R	τ_R	u_R	c_R	t_R	d'_R	s'_R	b'_R
Q	-1	-1	-1	$2/3$	$2/3$	$2/3$	$-1/3$	$-1/3$	$-1/3$
y	-2	-2	-2	$4/3$	$4/3$	$4/3$	$-2/3$	$-2/3$	$-2/3$

So that determines how the fermions couple to the gauge fields. But note that a mass term for fermions requires a coupling of the right and left handed components, $m\bar\psi\psi$ is the product of the right-handed piece with the left-handed one. And as these behave differently under SU(2), we cannot have such a term in the Lagrangian, and all our fermions must start out massless. We will see how we can get masses, however, from interaction couplings $\bar\psi\psi\phi$ as long as we extract the part of $\bar\psi\psi$ which transforms like $\phi^\dagger$. Then the vacuum expection part of the Higgs field becomes a mass, but the remaining part is a coupling of the fermions to the physical Higgs particle.

We will also see that there is an undetermined misalignment between the hadronic isospin doublets of different flavored quarks and their weak isospin. This will lead to the CKM matrix, a complex parameter, and CP violation.