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Lecture 12

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Wick's Theorem and Feynman Diagrams Copyright©2006 by Joel A. Shapiro

Last time we found that we could express any vacuum expectation value of the Time-ordered product of the full, interacting fields as

$$\langle \Omega | T\left(\prod_{j} \phi(x_{j})\right) | \Omega \rangle = \lim_{T \to \infty(1-i\epsilon)} \frac{\langle 0 | T\left(\prod_{j} \phi_{I}(x_{j})\right) e^{-i \int_{-T}^{T} dt H_{I}(t) dt} | 0 \rangle}{\langle 0 | Te^{-i \int_{-T}^{T} dt H_{I}(t) dt} | 0 \rangle}$$

Read Peskin and Schroeder, pages 88-99.

On page 92:

In order to do something different, let's consider a different piece in the contraction of

$$\langle 0 | \phi(x)\phi(y)\frac{1}{3!}(-i\mathcal{H}_I(z))(-i\mathcal{H}_I(w))(-i\mathcal{H}_I(u)) | 0 \rangle$$

given by



For the contraction with $\phi(x)$, we get the same diagram if we contract with any of the four ϕ 's in any of the three internal vertices¹, for a factor of 12. Call that vertex the z vertex. Then $\phi(y)$ can be contracted with any of the three remaining ϕ 's on the z vertex² So we multiply by 3. Then the first of the remaining two ϕ 's from the z vertex can be contracted with any of the four ϕ 's in either of the remaining internal vertices³, so this gives us a factor of 8, and we choose to call the attached vertex w. The last ϕ on z must then be contracted with the remaining unattached vertex⁴, u to any of the 4 ϕ 's. There remains the three ϕ 's on the w vertex which can be attached to the three remaining ϕ 's on the u vertex in 3! ways⁵. So the symmetry factor for this diagram is

$$S = \frac{12 \times 3 \times 8 \times 4 \times 3!}{3! \times (4!)^3} = \frac{1}{12}.$$

¹The other possibility, contracting with $\phi(y)$, would give a disconnected diagram (see p. 96) consisting of a noninteracting propagator times a product of disconnected pieces.

²Of course it could also be contracted with a different vertex, but that would not give rise to the same Feynman diagram. There are 8 equivalent ϕ 's for that possibility. Then there are several distinct ways the next ϕ on the z vertex can be contracted, several of which lead to the diagram explored in the book on p. 92.

³Or it could be contracted with the other $z \phi$, which would give the first graph on page 93 multiplied by vacuum bubbles.

 $^{^4 \}mathrm{Or}$ to another of the $w~\phi$'s, in which case we could get other graphs not shown in the book.

 $^{^5\}mathrm{Or}$ perhaps there is only one $w{-}u$ connection, in which case the diagram looks like a mouse.