Physics 507 Homework Solution #2
Due: Thursday, Sept. 16, 2010

1. Analyze the errors in Euler's method.
By the mean value theorem for the differentiable function $\dot{x}(t)$, $\dot{x}(t) = \dot{x}(t_i) + (t - t_i)\ddot{x}(\xi)$ for some point $\xi \in (t_i, t)$. Integrating this, we find that $x(t_{i+1}) - x(t_i) = \Delta t\, \dot{x}(t_i) + E$, where the error E has a magnitude $|E| \leq \frac{1}{2}(\Delta t)^2 \max|\ddot{x}(\xi)|$. The number of steps to cover a finite interval t with steps of size Δt is $t/\Delta t$, so the maximum cumulative error is less than $\frac{1}{2} t \max|\ddot{x}(\xi)| \times \Delta t \propto \Delta t$.

2. Numerical calculations with varying Δt.
I used the program given in the text, looking for the maximum of x for times near $t = 2\pi$ (one period). Here are my results for the maximum excess amplitude, as a function of Δt.

Δt	excess
.1	.368106
.04	.133986
.02	.064878
.01	.031925
.003	.009471
.001	.003145