

Midterm Exam

Possibly useful information

$$Y_{\ell m}(\theta, \phi) = \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_{\ell}^m(\cos \theta) e^{im\phi}$$

$$P_{\ell}^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_{\ell}(x), \quad \text{so } P_{\ell}^0(0) = P_{\ell}(0),$$

$$P_n(\pm 1) = (\pm 1)^n, \quad P_{2n}(0) = (-1)^n \frac{(2n-1)!!}{(2n)!!}, \quad P_{2n+1}(0) = 0$$

$$P_{2n}^1 = 0, \quad P_{2n+1}^1 = (-1)^n \frac{(2n+1)!!}{(2n)!!}, \quad Y_{\ell, -m}(\theta, \phi) = (-1)^m Y_{\ell, -m}^*(\theta, \phi).$$

Zeros of Bessel functions and their derivatives

J_0	J_1	J_2	J'_0	J'_1	J'_2	j_0	j_1	j_2	j'_0	j'_1	j'_2
2.405	3.832	5.136	3.832	1.841	3.054	3.142	4.493	5.763	2.798	4.222	5.523
5.520	7.016	8.417	7.016	5.331	6.706	6.283	7.725	9.095	6.121	7.587	8.971
8.654	10.173	11.630	10.173	8.536	9.970	9.425	10.904	12.323	9.318	10.810	12.236

N_0	N_1	N_2	N'_0	N'_1	N'_2	n_0	n_1	n_2	n'_0	n'_1	n'_2
0.894	2.197	3.384	1.961	3.534	4.882	1.571	2.798	3.960	2.798	4.222	5.523
3.958	5.430	6.794	5.337	6.868	8.287	4.712	6.121	7.452	6.121	7.587	8.971
7.086	8.596	10.023	8.538	10.073	11.530	7.854	9.318	10.716	9.318	10.809	12.236

Spherical Bessel functions

$$j_0(x) = \frac{\sin x}{x}, \quad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}, \quad n_0(x) = -\frac{\cos x}{x}, \quad n_1(x) = -\frac{\cos x}{x^2} - \frac{\sin x}{x}$$

For small x

$$j_{\ell}(x) \rightarrow \frac{x^{\ell}}{(2\ell+1)!!} \left(1 - \frac{x^2}{2(2\ell+3)} + \dots \right)$$

$$n_{\ell}(x) \rightarrow -\frac{(2\ell-1)!!}{x^{\ell+1}} \left(1 - \frac{x^2}{2(1-2\ell)} + \dots \right)$$

For $x \gg \ell$ (and $x \gg 1$)

$$j_{\ell}(x) \rightarrow \frac{1}{x} \sin \left(x - \frac{\ell\pi}{2} \right)$$

$$n_{\ell}(x) \rightarrow -\frac{1}{x} \cos \left(x - \frac{\ell\pi}{2} \right)$$