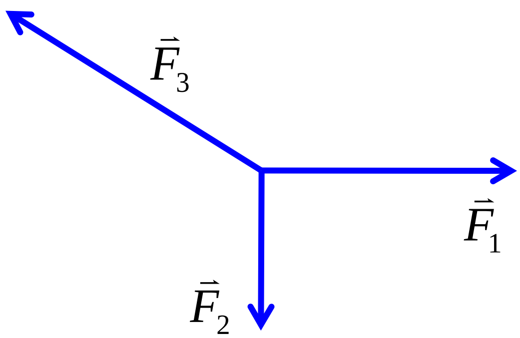
Circular Motion and Circular Dynamics

Review of Previous Lecture:

- Newton's First Law:

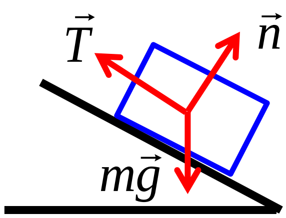


$$\sum \vec{F} = 0 \Rightarrow \vec{a} = 0$$

$$(v = \text{const.}; \quad v = 0)$$

$$\sum_i F_{xi} = 0; \quad \sum_i F_{yi} = 0$$

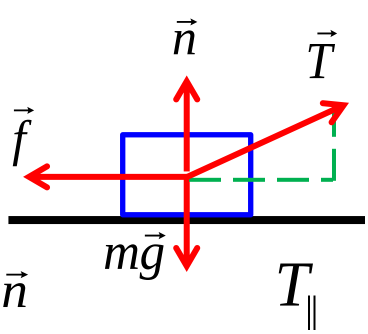
- Newton's Second law



$$\sum \vec{F} = m\vec{a}$$

$$\sum_i F_{xi} = ma_x; \quad \sum_i F_{yi} = ma_y$$

- Resistive Force (I): Friction



$\vec{f} \perp \vec{n}$

$$f_k = \mu_k n$$

$$(f_s)_{\max} = \mu_s n$$

$$\text{If} \quad v = 0 \ \rightarrow \ f_s = T_\parallel$$

$$\text{If} \quad v = \text{const.} \ \rightarrow \ f_k = T_\parallel$$

i-Clicker

A lightweight crate (A) and a heavy crate (B) are side-by-side on a frictionless horizontal surface. You are applying a horizontal force F to crate A. Which of the following forces *should* be included in a free-body diagram for crate B?

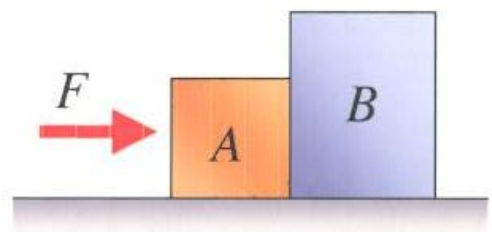
A. the weight of crate B

B. the force of crate B on crate A

C. the force F that you exert

D. the acceleration of crate B

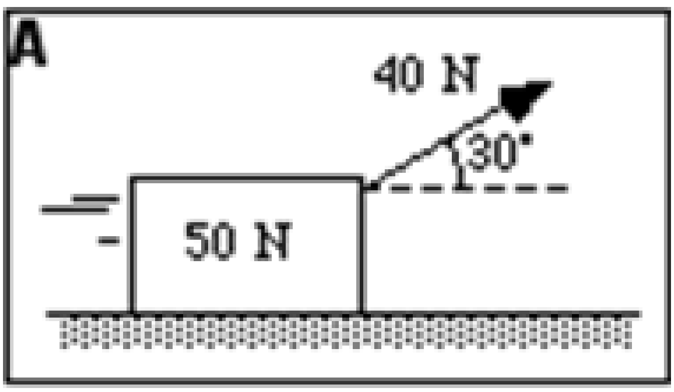
E. two or more of the above forces

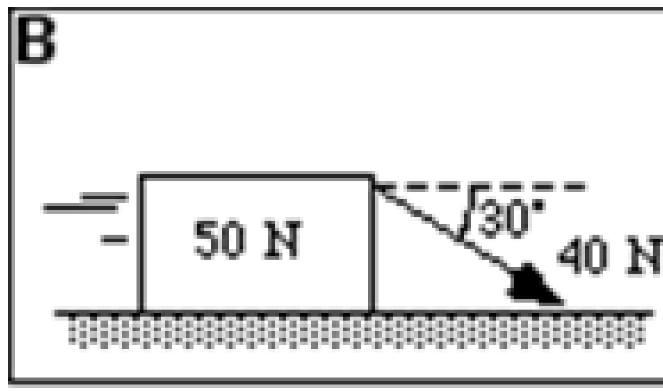


Only forces that at on crate B
are included in the free body diagram
for crate B

i-Clicker

A box weighing 50 N is pulled with a force of 40 N. The box is sliding to the right and is subject to friction. In case A, the pulling rope is at an upward angle of 30°. In case B, the force is at a downward angle of 30°.





A. The friction force in case A is greater than in case B.

B. The friction force is the same in both cases.

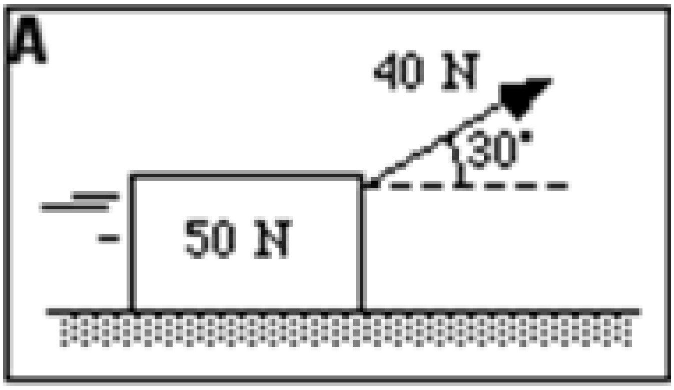
C. The friction force in case A is less than in case B.

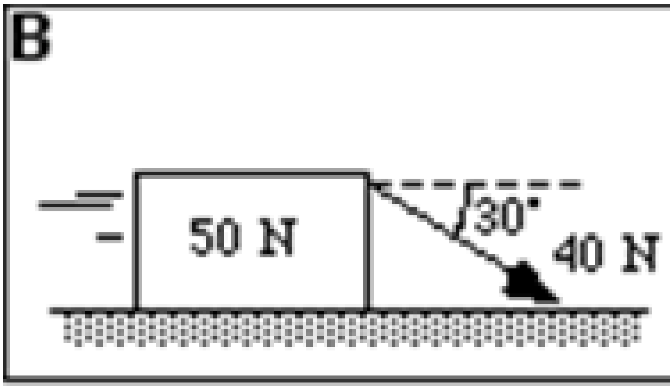
D. The friction forces cannot be compared without knowing the kinetic friction coefficient.

E. The friction forces cannot be compared without knowing the speed and acceleration.

i-Clicker

A box weighing 50 N is pulled with a force of 40 N. The box is not moving due to static friction. In case A, the pulling rope is at an upward angle of 30°. In case B, the force is at a downward angle of 30°.





A. The friction force in case A is greater than in case B.

B. The friction force is the same in both cases.

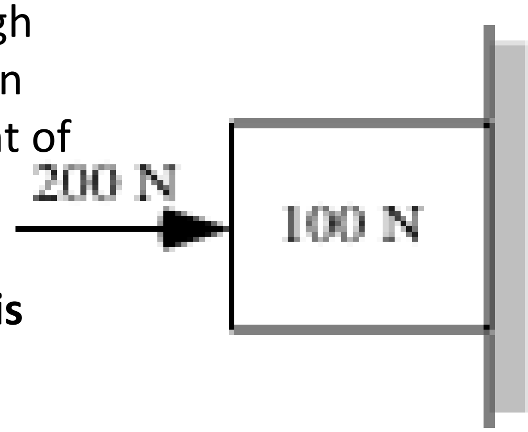
C. The friction force in case A is less than in case B.

D. The friction forces cannot be compared without knowing the static friction coefficient.

E. Static friction is not large enough to keep the box from sliding.

i-Clicker

A woman is applying a horizontal force of 200 N on a 100 N box, trying to pin it against a rough wall. The coefficient of static friction between the box and the wall is 0.6 and the coefficient of kinetic friction is 0.55.



Which statement about the frictional force is correct ?

A.) *The frictional force is 60 N since the box will not be moving and the coefficient of static friction is 0.6.*

B.) The frictional force is 100 N upward since it balances the downward weight of 100 N.

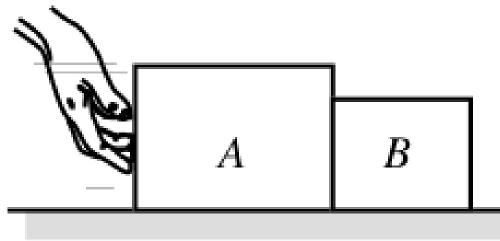
C.) The frictional force will be 120 N since the box is not moving and the normal force is 200 N.

D.) The box will start sliding and the frictional force will be 110 N.

E.) None of the above

i-Clicker

A student pushes horizontally on two blocks. The blocks are moving to the right and are slowing down. Block *A* has more mass than block *B.* There is friction between the blocks and the table.



Which of the following statements about the net forces is true?

A) The net force on block A is *equal to* the net force on block B.

B) The net force on block *A* points *to the right* and is *greater than* the net force on block *B*.

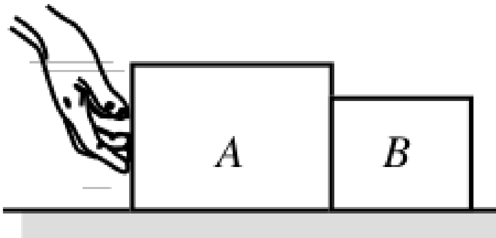
C) The net force on block *A* points *to the left* and is *greater than* the net force on block *B*.

D) The net force on block *A* points *to the right* and is *less than* the net force on block *B*.

E) The net force on block *A* points *to the left* and is *less than* the net force on block *B*.

i-Clicker

A student pushes horizontally on two blocks. The blocks are moving to the right and are slowing down. Block *A* has more mass than block *B.* There is friction between the blocks and the table.



Which of the following statements about the magnitude of the forces is true?

A) The force that block *A* exerts on block *B* is *greater than* the force that block *B* exerts on block *A.*

B) The force that block *A* exerts on block *B* is *less than* the force that block *B* exerts on block *A.*

C) The force that block *A* exerts on block *B* is *equal to* the force that block *B* exerts on block *A.*

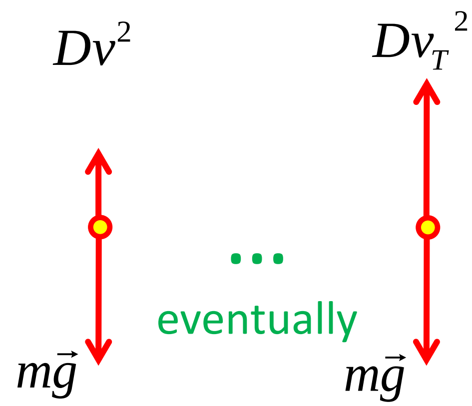
D) We cannot compare the forces unless we know how fast the blocks are slowing down.

Other Resistive Forces: Drag (in fluids) :

- Laminar Flow (oil, syrup): $f = kv$
- Turbulent Flow (air, water): $f = Dv^2$

Example: Skydiver

Free body Diagram:

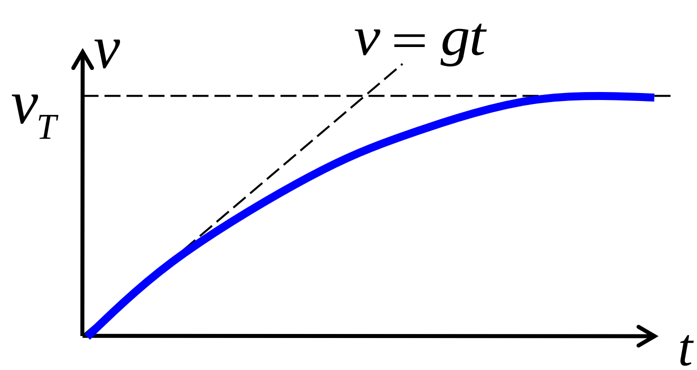


v

Terminal velocity:

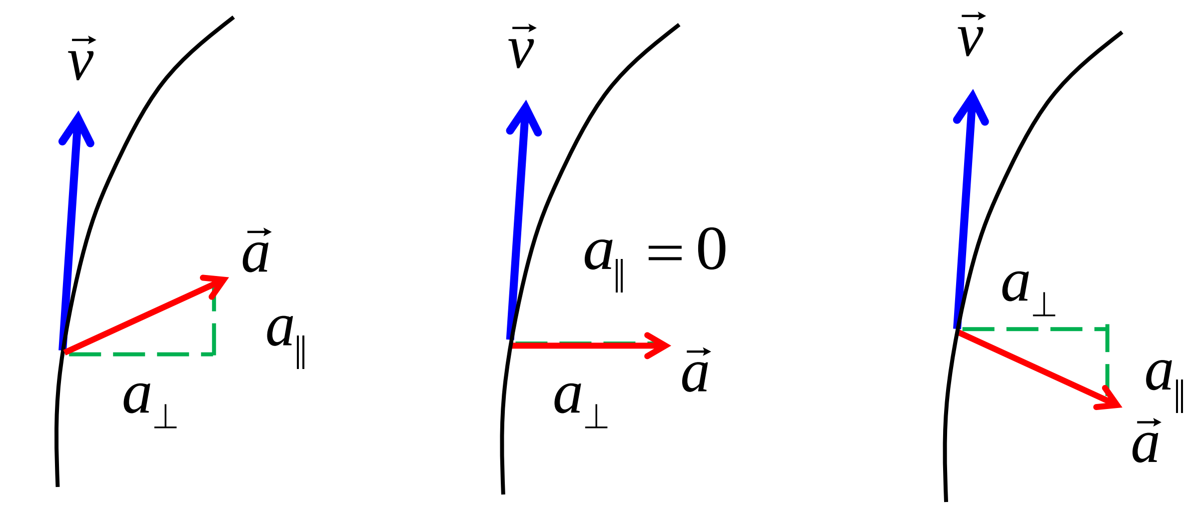
$$Dv_T^{\ 2} = mg$$

$$v_T = \sqrt{\frac{mg}{D}}$$

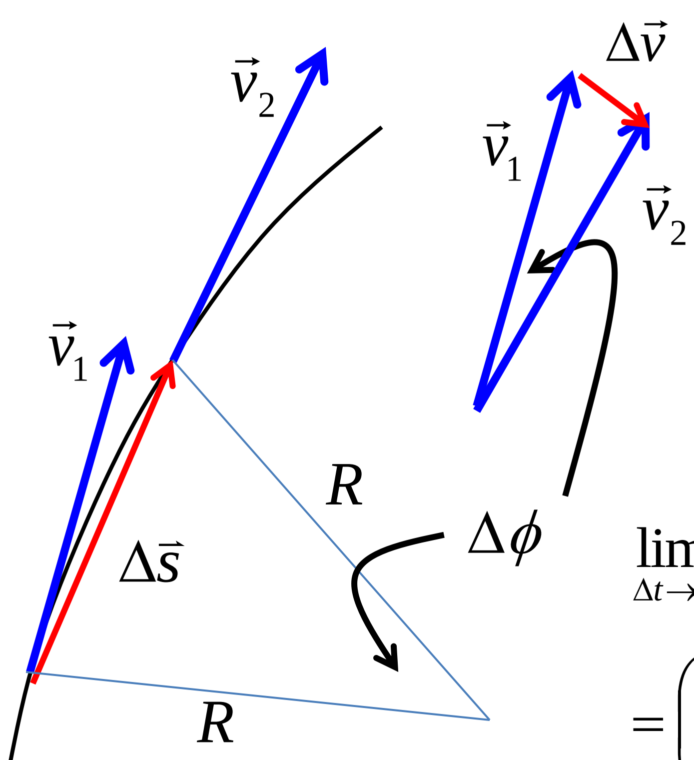


NOTE:
v_T depends on mass

Recall the relationship of $\vec{v}$ and $\vec{a}$ along a curved path



v increasing	v constant	v decreasing
$\vec{v}$ changing direction	$\vec{v}$ changing direction	$\vec{v}$ changing direction



$$\frac{|\Delta\vec{s}|}{R} = \frac{|\Delta\vec{v}|}{v}$$

$$\frac{|\Delta\vec{v}|}{\Delta t} = \frac{v}{R}\frac{|\Delta\vec{s}|}{\Delta t}$$

$$\lim_{\Delta t\to 0}\frac{|\Delta\vec{v}|}{\Delta t} = a = \left(\frac{v}{R}\right)\lim_{\Delta t\to 0}\frac{|\Delta\vec{s}|}{\Delta t}$$

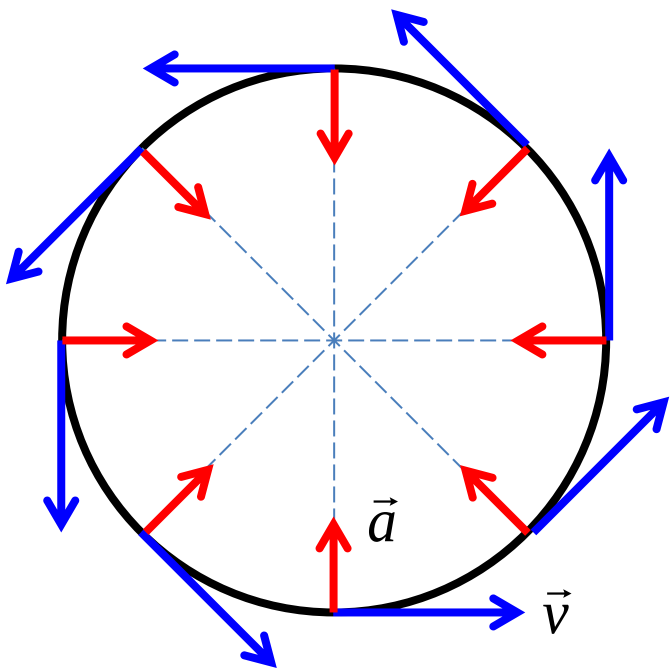
$$= \left(\frac{v}{R}\right)v \Rightarrow \boxed{a = \frac{v^2}{R}}$$

Uniform Circular Motion

v constant

$\vec{v}$ always changing direction

(always tangent to circle)



a constant

$\vec{a}$ always changing direction

(always pointing to center of the circle)

Write $a = a_{\text{rad}}$ radial acceleration

$$a_{rad} = \frac{v^2}{R}$$

Centripetal acceleration ("seeking center")

Particle travels full circumference in period T

$$v = \frac{2\pi R}{T}$$

$$a_{rad} = \left(\frac{2\pi R}{T}\right)^2 \frac{1}{R} \Rightarrow \boxed{a_{rad} = \frac{4\pi^2 R}{T^2}}$$

Example: GPS satellites orbit the earth twice per day at a constant altitude of 20,200 km. What is a_{rad} ?

$$T = (12\text{ h})(60\text{ min/hr})(60\text{ s/min})$$
$$= 43{,}200\text{ s} = 4.32 \times 10^4\text{ s}$$

$$v = 2\pi R / T$$

$$R = 26{,}600\text{ km} = 2.66 \times 10^7\text{ m}$$

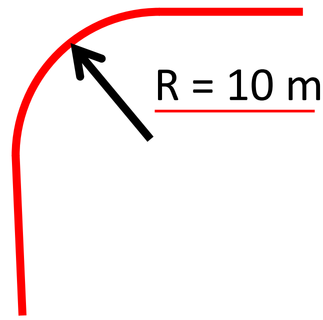
$$v = 2\pi(2.66 \times 10^7\text{ m}) / (4.32 \times 10^4\text{ s})$$
$$= 3.87 \times 10^3\text{ m/s}$$

$$a = v^2/R = 0.56\text{ m/s}^2$$

Example: A cop pulls you over after making a fast right turn in a 25 mph zone. He wants to give you a ticket for speeding.

But you say *NO!* My tires do not allow acceleration greater than g without skidding, therefore I could not have been speeding!

Should the officer give you the ticket?



$$a_r = v^2/R \rightarrow v^2 = a_r R$$

$$v^2 = (10\ \mathrm{m/s^2})(10\ \mathrm{m}) = (100\ \mathrm{m^2/s^2})$$

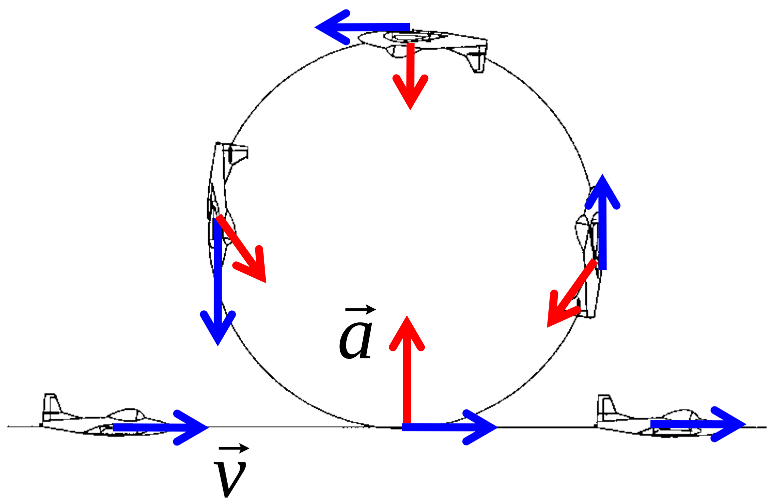
$$v = 10\ \mathrm{m/s}$$

$$1\ \mathrm{m/s} = 2.3\ \mathrm{mph}$$

$$\rightarrow 10\ \mathrm{m/s} = 23\ \mathrm{mph}$$

"Better luck next time copper."

Circular Motion is not always uniform!!!

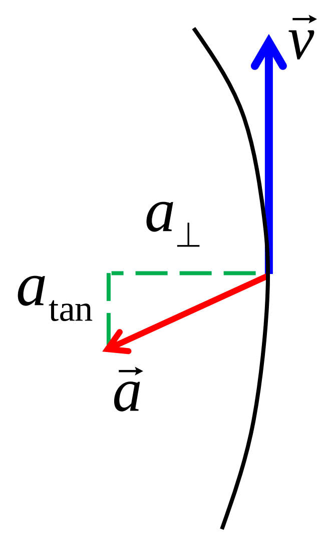


Path is circular <u>but</u>: a not constant

v not constant

$\vec{a}$ not $\perp$ to $\vec{v}$

$$a_{\tan} = \frac{d|\vec{v}|}{dt} = \frac{dv}{dt}$$



$$a_{\perp} == a_{rad} = \frac{v^2}{R}$$

Instantaneous speed (not constant)

R is constant (circular path)

Dynamics of Uniform Circular Motion

Recall:

$$a_{rad} = \frac{v^2}{R} \quad ; \quad T = \frac{2\pi R}{v} \quad ; \quad a_{rad} = \frac{4\pi R}{T^2}$$

Newton's 2nd Law:

Must apply a force to cause acceleration

$$\boxed{F_c = ma_{rad} = \frac{mv^2}{R}}$$ Centripetal Force

$\vec{a}$ points toward the center of the circle

$\vec{F}_c$ points towards the center of the circle

To produce uniform circular motion
something must provide the
centripetal force

i-Clicker

Four identical small cylinders rest on a circular horizontal turntable at the positions shown in the diagram below. The turntable is rotating clockwise at a constant angular speed. **Rank the magnitude of the net force on the cylinder at the positions on the turntable indicated in the diagram.**

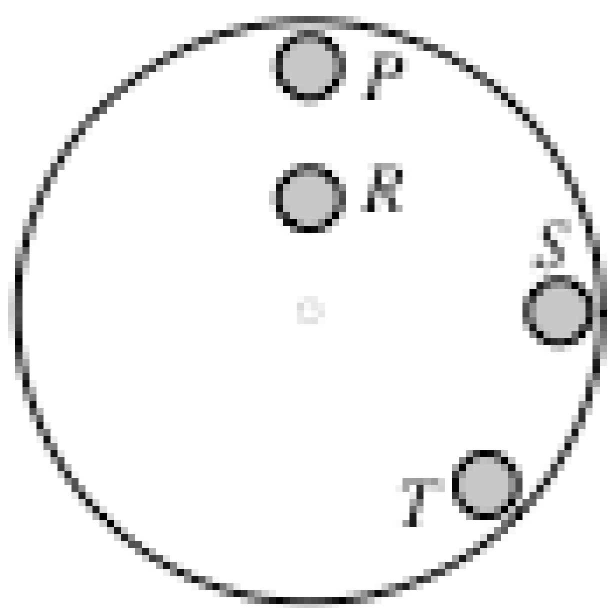
A. R > P = S = T

B. P = S = T > R

C. P = R = S = T > 0

D. P = R = S = T = 0

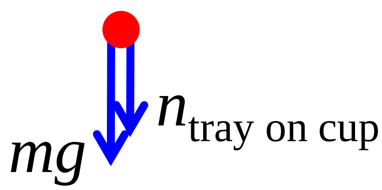
E. Cannot be determined without knowing rotational speed.

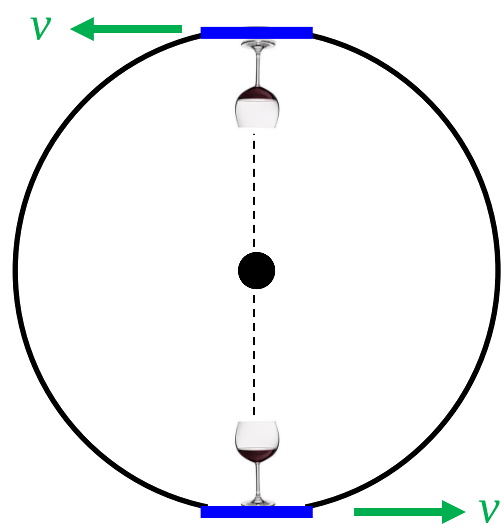


Circular Motion in The Vertical Plane:

Consider forces on the glass at the top of the circle

Free body diagram
for the glass





$$\sum F_r = mg + n = ma_r$$

(positive because toward the center)

$$\rightarrow \quad ma_r = n + mg$$

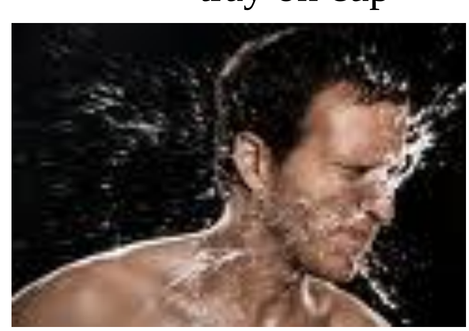
$$\rightarrow \quad n = m(a_r - g)$$

But $\quad a_r = v^2/R$

$$\rightarrow \quad n = m(v^2/R - g)$$

So, if slow down until: $v^2/R = g$ then $n_{\text{tray on cup}} = 0$

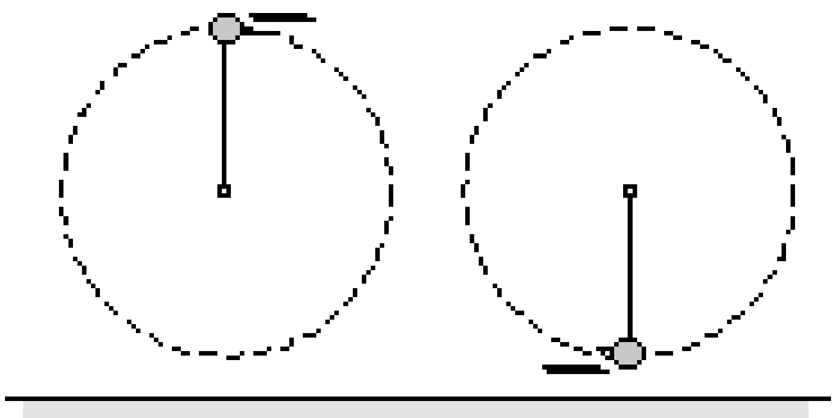
At which point glass goes flying…



→ GET A TOWEL!!!

i-Clicker

A ball with a weight of 2 N is attached to the end of a cord of length 2 meters. The ball is whirled in a vertical circle counterclockwise as shown below. The tension in the cord at the top of the circle is 7 N and at the bottom it is 15 N. (Do not assume that the speed of the ball is the same at these points.) **Which is correct?**



A. The net force on the ball at the top position is 9 N.

B. The acceleration has the same value at the top and at the bottom.

C. The net force on the ball at the bottom position is 17 N.

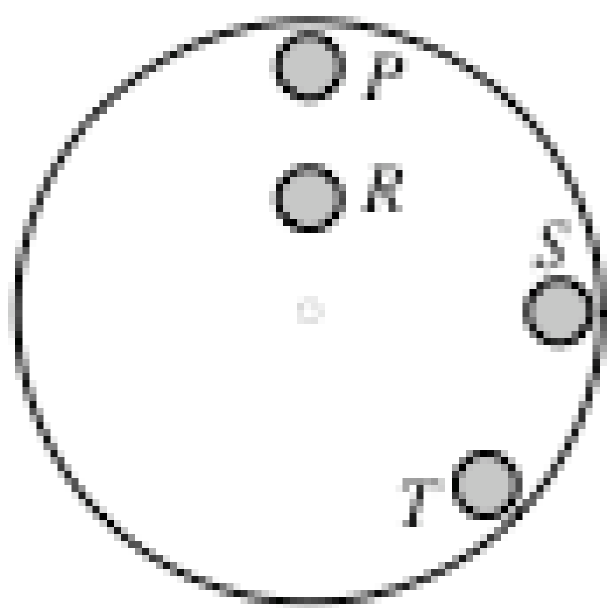
D. The net force on the ball at the top position is 7 N.

E. The value of the net force cannot be determined w/o knowing the speed.

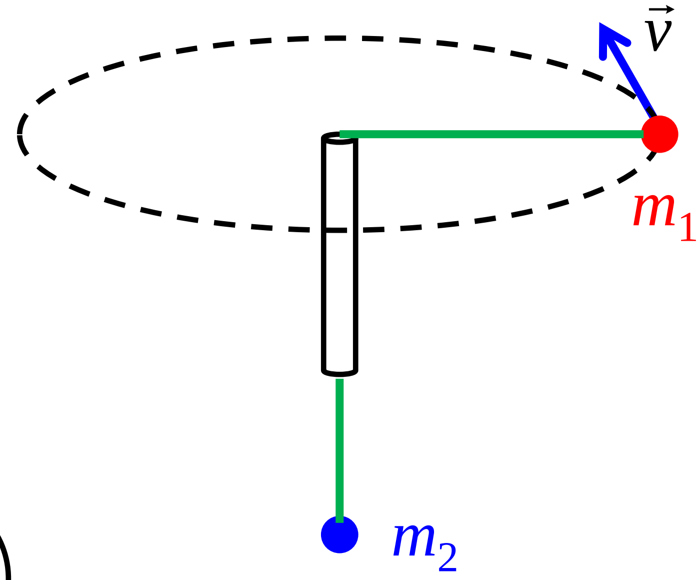
i-Clicker

Four identical small cylinders are attached to a circular vertical turntable at the positions shown in the diagram below. The turntable is rotating clockwise at a constant angular speed. **Rank the magnitude of the net force on the cylinder at the positions on the turntable indicated in the diagram.**

A. R > P = S = T
B. P = S = T > R
C. P = R = S = T
D. P > R > S > T
E. Cannot be determined without rotational speed.

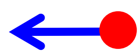


Consider m_1 in uniform circular motion.



What provides centripetal force?

$$\sum_i F_{x_i} = \frac{m_1 v^2}{R} = T$$

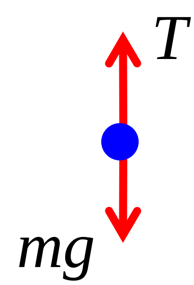


Free body diagram for m_1

If $\dfrac{m_1 v^2}{R} > m_2 g$

Then $T > m_2 g \Rightarrow$

m_2 <u>*accelerates*</u> vertically upward.



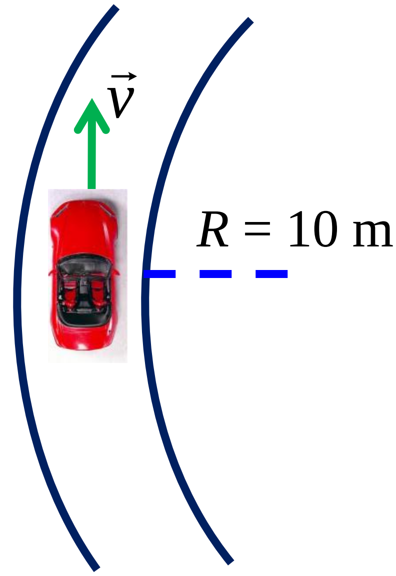
Free body diagram for m_2

$$\sum_i F_{y_i} = 0 = T - m_2 g$$

$$T = m_2 g$$

Is there something missing here??

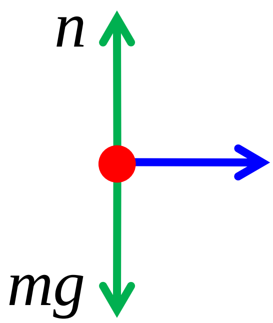
Lets return to the fast right turn.



(topview)

What is the minimum μ_s
to ensure no skidding?

(v = 10 m/s)



$$F_c = \frac{mv^2}{R} = F_s = \mu_s n$$

Free body diagram for car
(<u>rear view</u>)

$$\sum_i F_{y_i} = 0 = n - mg \Rightarrow n = mg$$

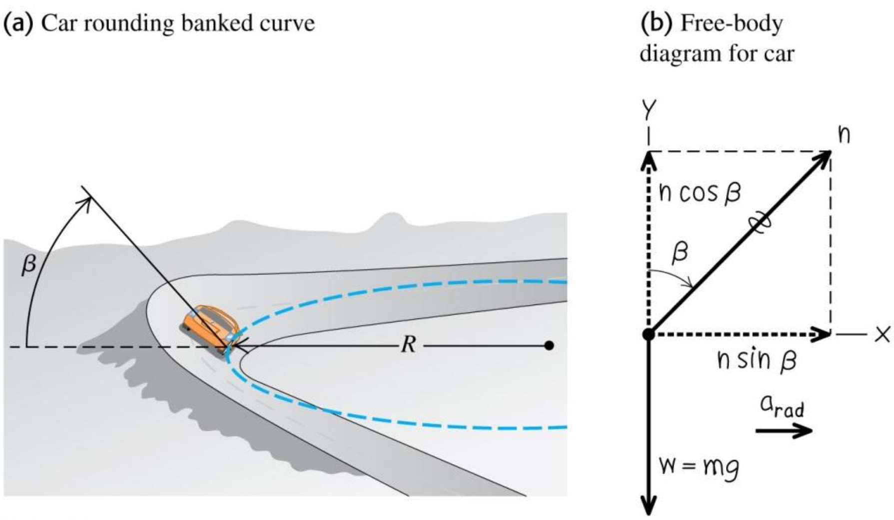
$$\sum_i F_{x_i} = ma_r = \left(f_s\right)_{\max} = \mu_s n$$

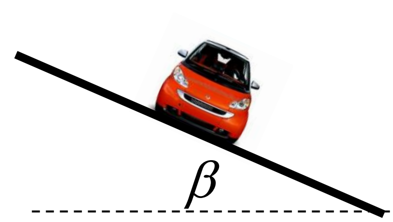
$$ma_r = \mu_s n = \mu_s mg \Rightarrow \mu_s = \frac{a_r}{g}$$

We had $a_r = g$, so we need $\mu_s = 1$

Suppose turn is <u>banked</u>.

What should angle β be so that car will turn at same speed on ice (μ_s=0) without slipping?

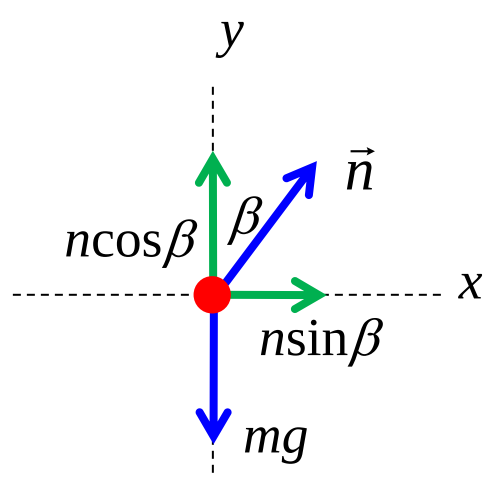




$$\sum F_x = \frac{mv^2}{R} = n\sin\beta = ma_r$$

$$\sum F_y = 0 = n\cos\beta - mg$$

$$\Rightarrow n\cos\beta = mg$$

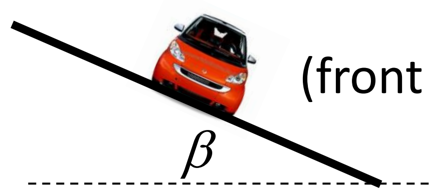


Free body diagram for car

$$\Rightarrow \frac{\cancel{n}\sin\beta}{\cancel{n}\cos\beta} = \frac{\cancel{m}a_r}{\cancel{m}g}$$

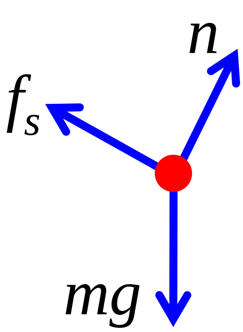
i-Clicker

A car moves at the fastest speed possible to negotiate a banked turn with the help of friction. Which is the correct free body diagram of the car?

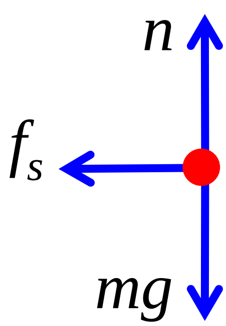


(front view)

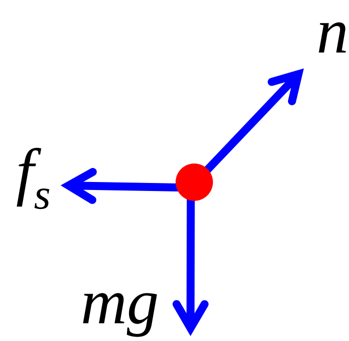
A



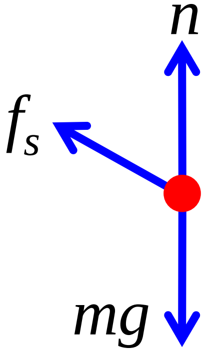
B



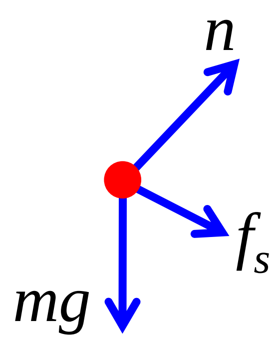
C



D



E



- n normal to surface
- mg vertically down
- f_s parallel to surface and inward