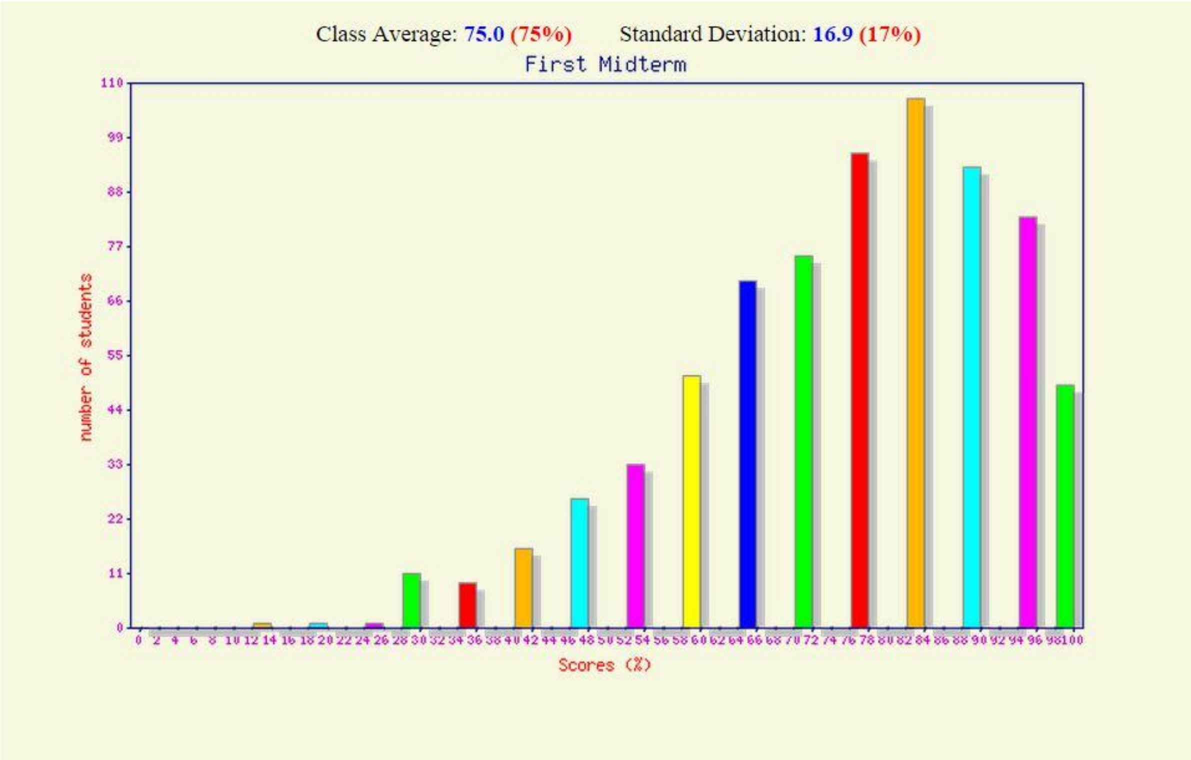
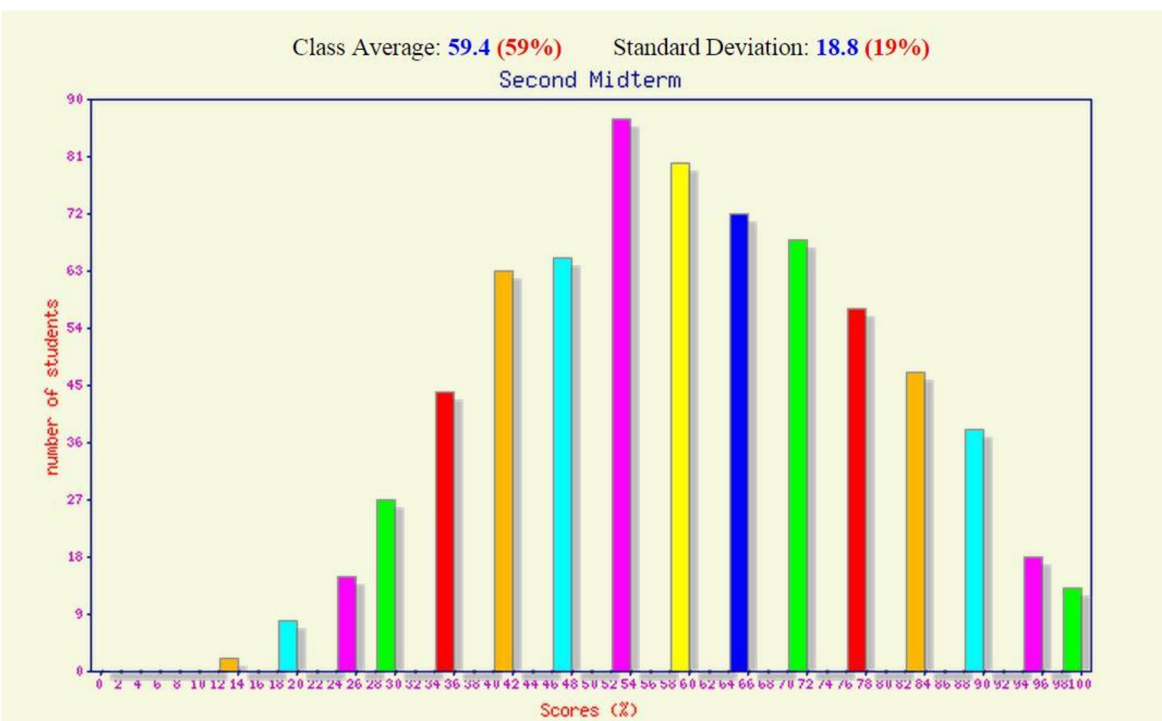
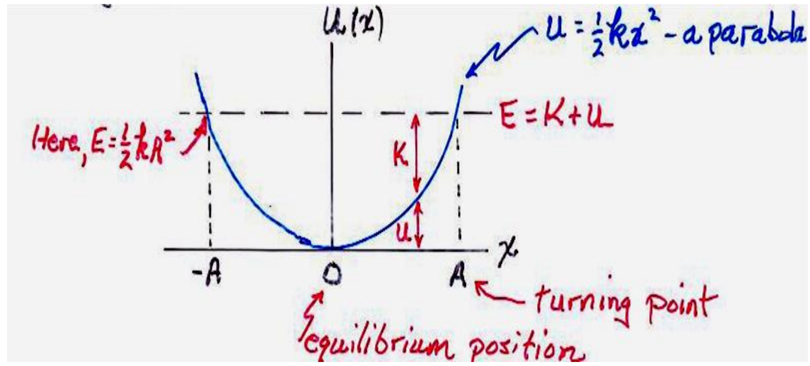
# Today's topic:
# IMPULSE AND MOMENTUM CONSERVATION



# Review of Last Week's Lecture

- Elastic Potential Energy:
  $x$: displacement from equilibrium
  $x = 0$: equilibrium position




- Work-Energy Theorem:

$$W_{tot} = W_g + W_{el} + W_{non-cons} = \Delta K = K_2 - K_1$$

$$W_{non-cons} = (K_2 + U_{g_2} + U_{el_2}) - (K_1 + U_{g_1} + U_{el_1})$$

Let $U = U_g + U_{el}$ = Potential Energy

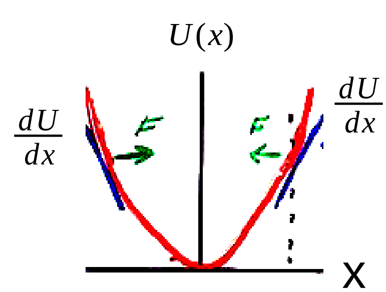
$E = K + U$ = Total Mechanical Energy

$$\Rightarrow W_{non-cons} = E_2 - E_1$$ (friction, drag, work done by muscles, etc.)

$$W_{non-cons} = 0 \quad \Rightarrow \quad E_2 = E_1$$ Conservation of mechanical energy

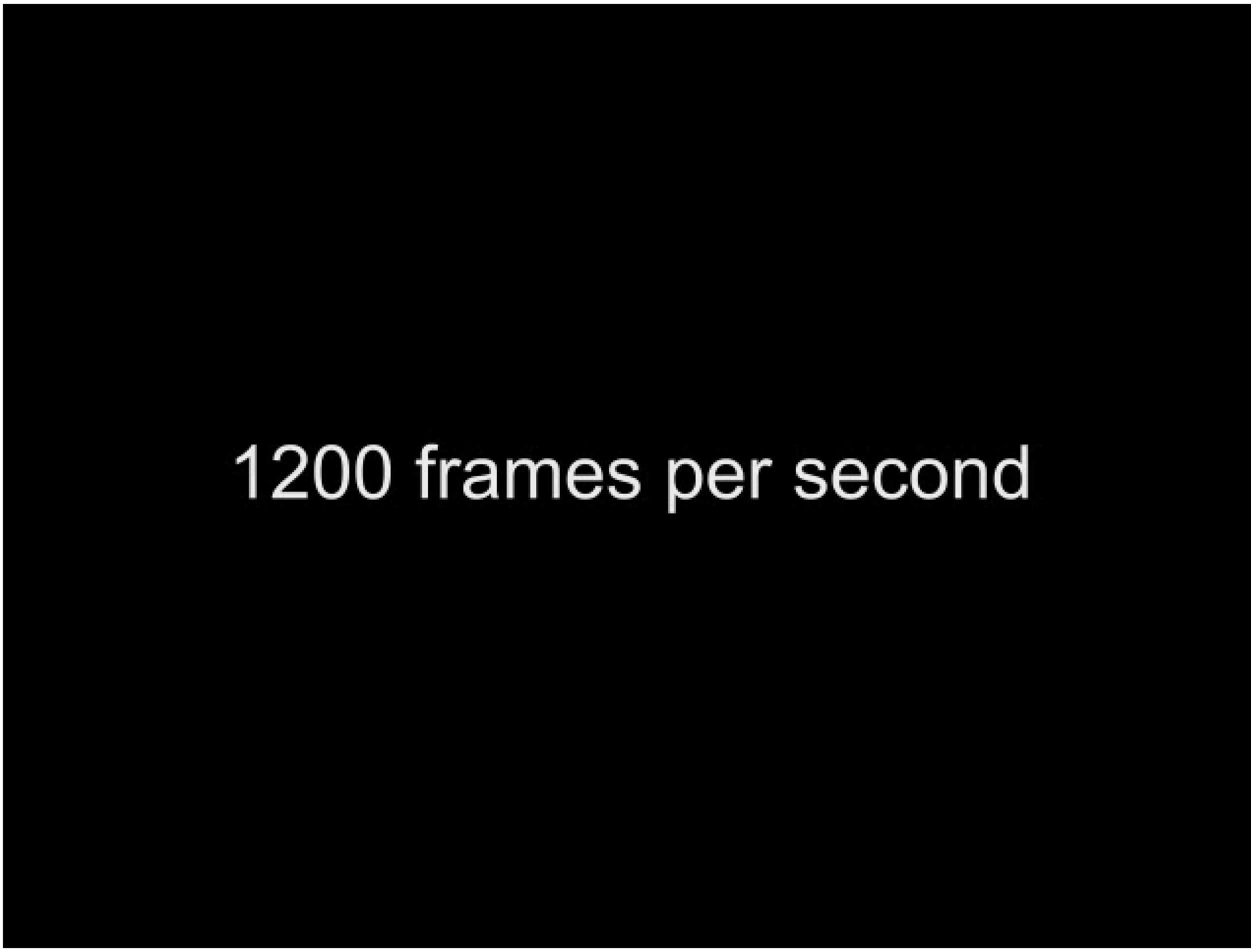
- Force and Potential Energy:

$$F_x = -\frac{dU}{dx}$$



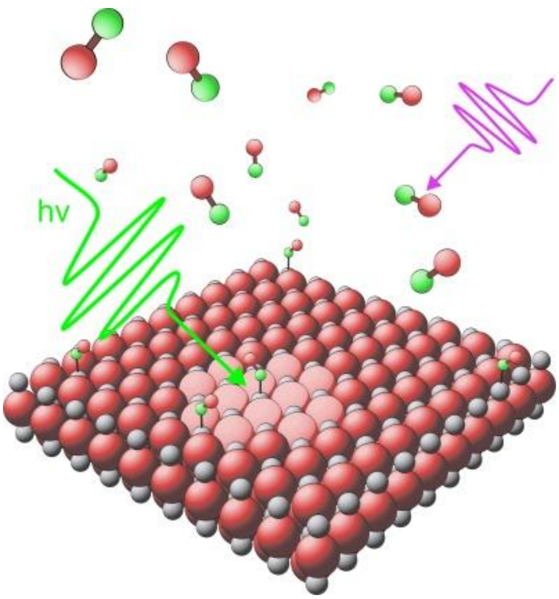
[Skate Park Animation](#)

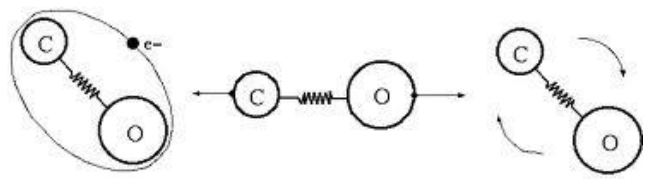
Slow motion video of last week's spring launch:

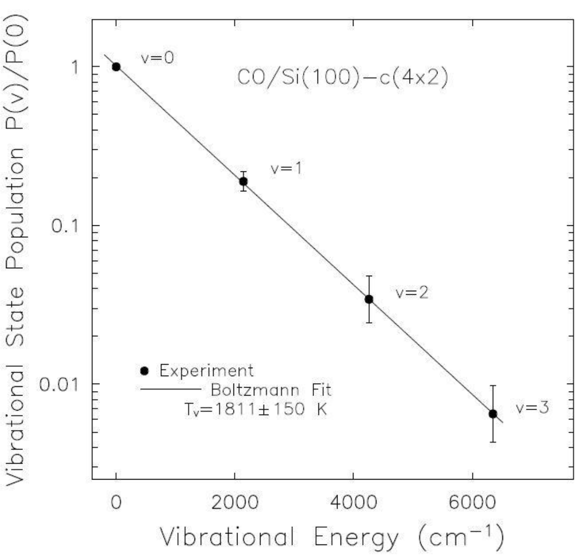


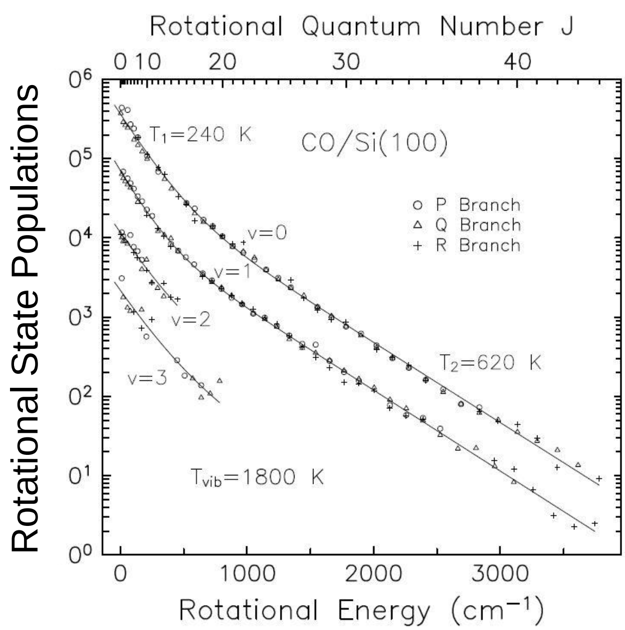

- What does the spring do, other than shooting up (and falling down)?
- Does vibrational/rotational motion store energy?
- What kind of energy?
- Did you account for this energy in last weeks workshop?
- Only about 2% of the total energy in vibration, much less  in rotation!

Spring launch may serve as model for molecules “desorbing” (i.e., detaching) from a surface:



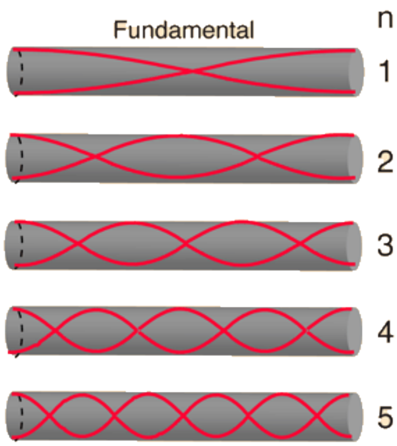




F. M. Zimmermann and W. Ho, Surface Science Reports 22, 127-247 (1995).

A spring can vibrate in many "normal modes":

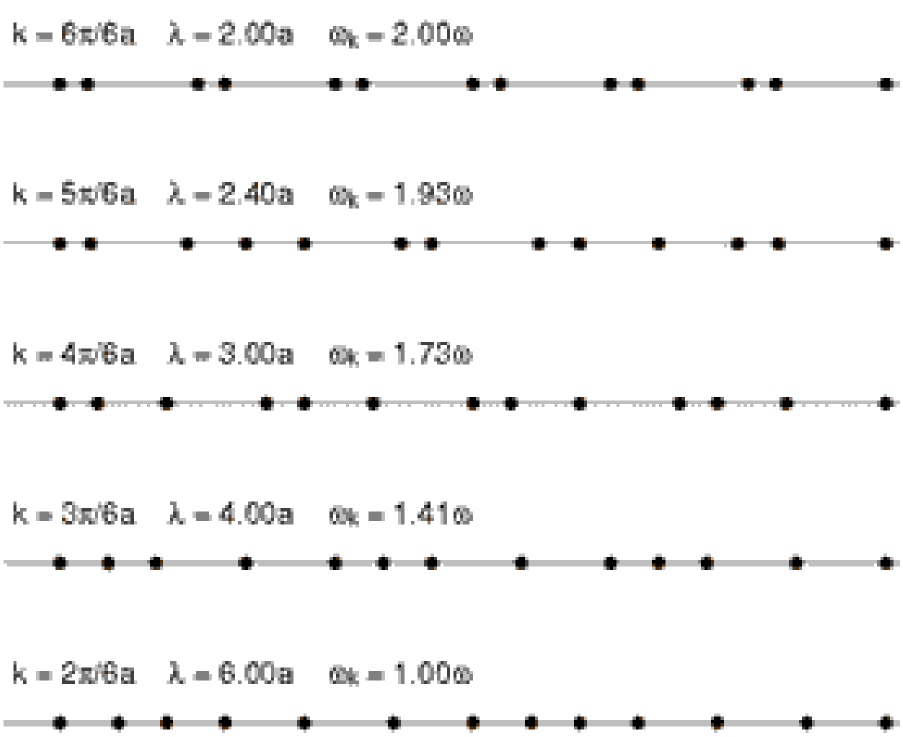



The higher the number of "nodes", the greater the vibrational frequency.

True not only for springs, but any solid!

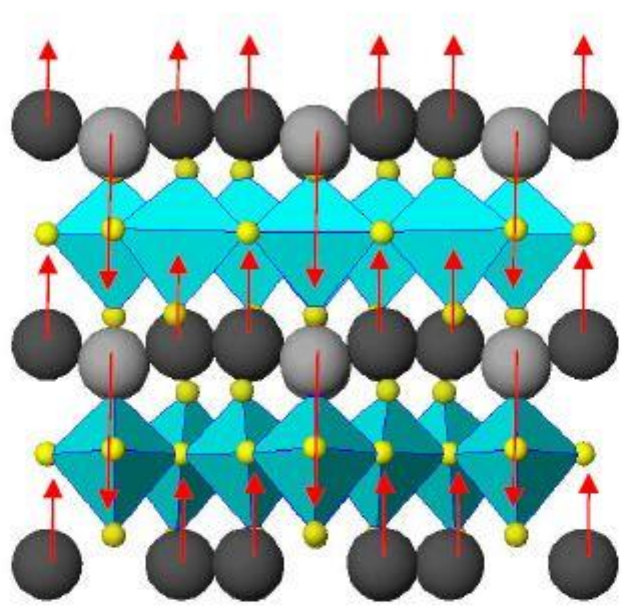
Do these modes continue to infinity (infinite # of nodes)?

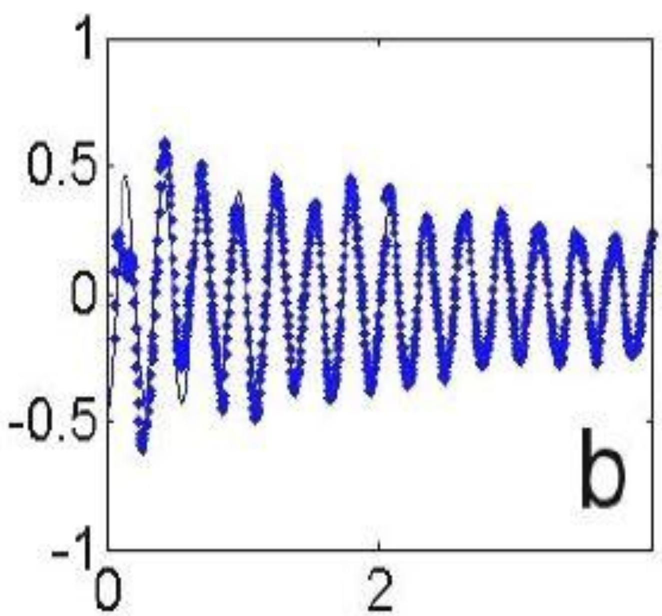
No, wavelength is limited by spacing between atoms:




Lattice Vibrations
or
"Phonon Modes"

Use femtosecond laser spectroscopy to measure phonon vibrations in LuMnO3 crystal:






Reflectivity change x $10^{-5}$

Pump-Probe Delay (picoseconds)

S. Lou, F. M. Zimmermann, R. A. Bartynski, N. Hur, and S. Cheong, Physical Review B 79, 214301 (2010).

# MOMENTUM & IMPULSE

NEWTON'S 2nd Law: $$\Sigma\vec{F} = m\vec{a}$$

Write differently:

$$\Sigma\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

- Define Momentum: $\vec{p} = m\vec{v}$ (Units: kg m/s = N s)

$$\Rightarrow \Sigma\vec{F} = \frac{d\vec{p}}{dt}$$

Net force = Rate of change of momentum

Consider this relationship further:

$$\Sigma\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = (\Sigma\vec{F})dt$$

$$\Rightarrow \int_{p_1}^{p_2} d\vec{p} = \int_{t_1}^{t_2} (\Sigma\vec{F})dt = \vec{p}_2 - \vec{p}_1$$

Define Impulse:

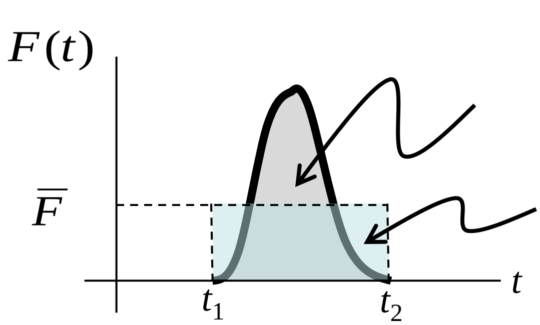
$$\vec{J} = \int_{t_1}^{t_2} (\Sigma\vec{F})dt = \vec{p}_2 - \vec{p}_1$$

Vector that equals change in momentum

Had: Work-Energy Theorem , now have:

**Impulse – Momentum Theorem:** $\vec{J} = \vec{p}_2 - \vec{p}_1$

Consider a variable force acting on an object from time $t_1$ to $t_2$ (e.g., basketball dribble)




$$J = \int_{t_1}^{t_2} F(t)\,dt$$

$$J = \bar{F} \times (t_2 - t_1)$$

Integral of actual force from $t_1$ to $t_2$ is equal to average force times interval $\Delta t = t_2 - t_1$

**COMPARISON:**
**MOMENTUM vs. KINETIC ENERGY:**

$\vec{p}$ is a vector ; KE is a scalar

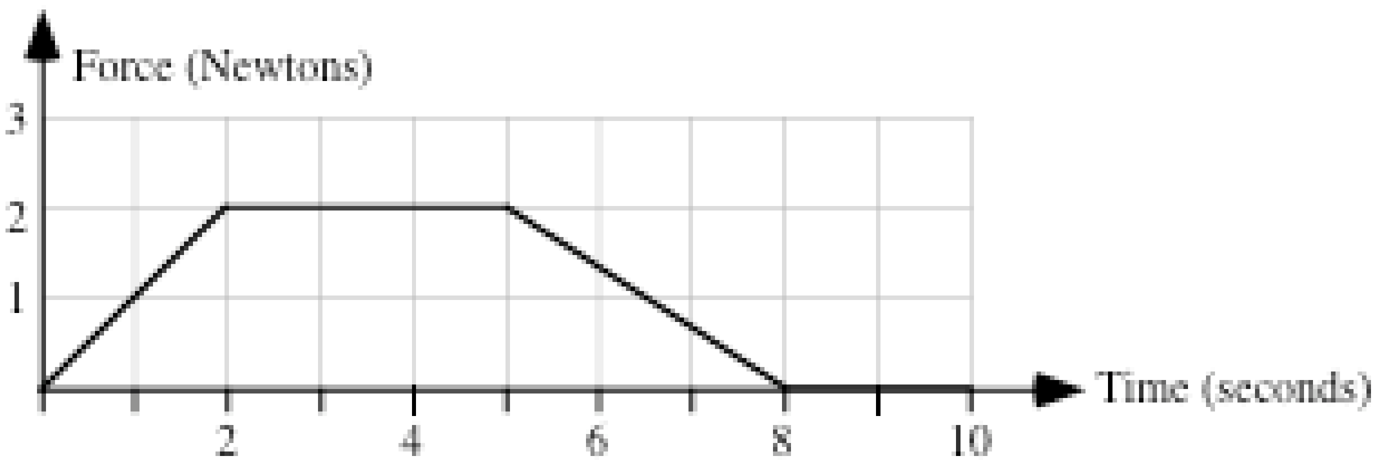
$p \propto v$ $\quad\quad$ $KE \propto v^2$

$\vec{p}$ related to time over which force acts !

$KE$ related to distance over which force acts !

# *i-Clicker*

A 10-kg box, initially at rest, moves along a frictionless horizontal surface. A horizontal force to the right is applied to the box. The magnitude of the force changes as a function of time as shown.




A. The impulse in the first 2 seconds is 2 kg·m/s
B. The impulse from 5 seconds to 8 seconds is -6 kg·m/s
C. The impulse in the first 2 seconds is 1 kg·m/s
D. The impulse from 2 seconds to 5 seconds is 0 kg·m/s
E. The impulse cannot be determines with the information given

# i-Clicker

You are testing a new car using crash test dummies. Consider two ways to slow the car from 90 km/h (56 mi/h) to a complete stop:

(i) You let the car slam into a wall, bringing it to a sudden stop.

(ii) You let the car plow into a giant tub of gelatin so that it comes to a gradual halt.

In which case is there a greater *impulse* of the net force on the car?

A. in case (i)

B. in case (ii)

C. The impulse is the same in both cases.

D. not enough information given to decide

E. I want 10 points subtracted from my grade

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

$$\vec{J} = 0 - \vec{p}_1$$

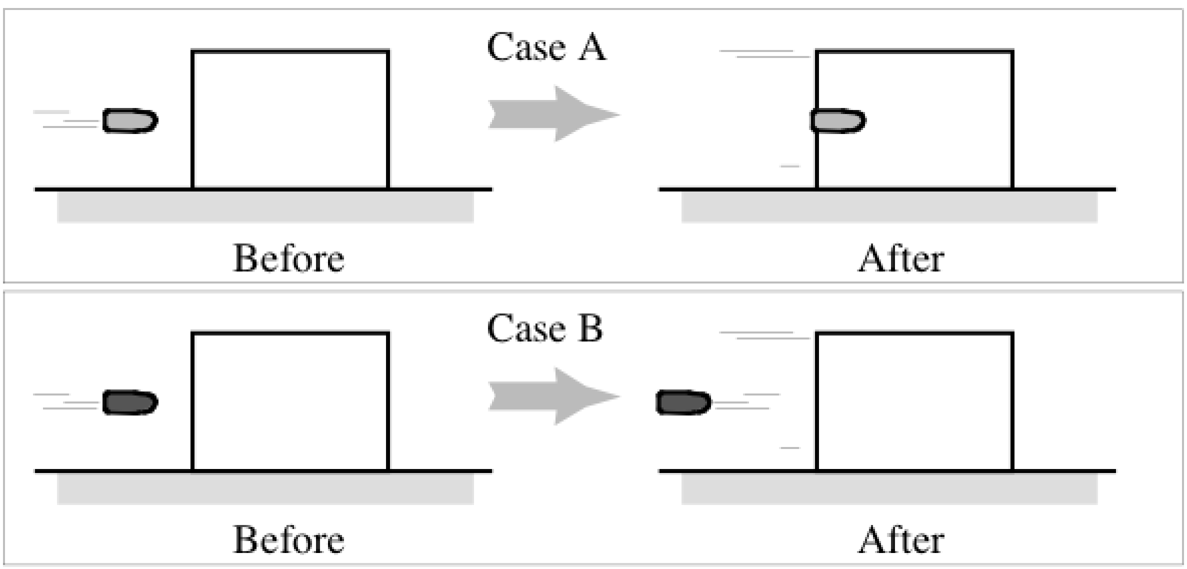
# *i-Clicker*

A 2-kg object accelerates in response to an applied force. During the 5-second interval that the force is applied, the object's velocity changes from 3 m/s east to 7 m/s west. Which is true about the magnitude of the impulse?


A. It equals 20 kg·m/s
B. It equals 8 kg·m/s
C. It equals 8/5 kg·m/s
D. It equals 4 kg·m/s
E. It cannot be found with the information given.

# ***i-Clicker***

In Case A, a metal bullet penetrates a wooden block. In Case B, a rubber bullet with the same initial speed and mass bounces off of an identical wooden block.
**Will the speed of the wooden block after the collision be *greater in Case A, greater in Case B,* or *the same in both cases*?**

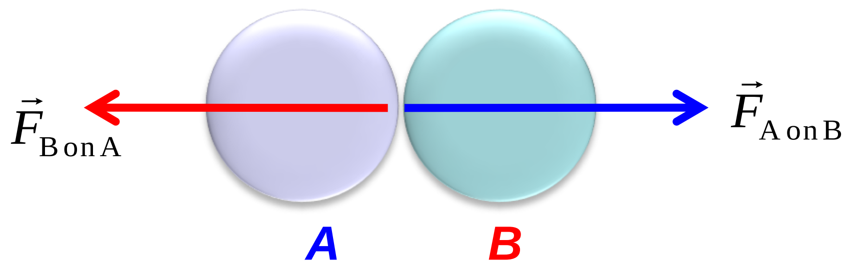



A. The speed will be greater in Case A because the metal bullet exerts a larger force on the block.
B. The speed will be greater in Case B because the bullet changes direction.
C. The speed will be the same in both cases because the bullets have the same mass and initial speed and give the block the same momentum.
D. Cannot be determined.

# CONSERVATION OF LINEAR MOMENTUM

Consider two isolated objects that interact only by their mutual force.

(No net external force)




$$\vec{F}_{B\,on\,A} = -\vec{F}_{A\,on\,B}$$ (Newton's 3rd Law)

$$\vec{F}_{A\,on\,B} + \vec{F}_{B\,on\,A} = 0$$

But… $\vec{F}_{B\,on\,A} = \dfrac{d\vec{p}_A}{dt}$ $\quad$ $\vec{F}_{A\,on\,B} = \dfrac{d\vec{p}_B}{dt}$

So: $\dfrac{d\vec{p}_A}{dt} + \dfrac{d\vec{p}_B}{dt} = \dfrac{d}{dt}(\vec{p}_A + \vec{p}_B) = 0$

For isolated system (no external forces) total linear momentum of the system is constant:

$$\vec{P} = \vec{p}_A + \vec{p}_B = \text{constant}$$

CONSERVATION OF LINEAR MOMENTUM

Conservation of momentum is valid for any number of particles interacting only with each other

(No External Forces)

$\vec{P} = \Sigma_i \vec{p}_i$ Is a vector quantity that is conserved

## EXAMPLE - physics of hockey:

A Ranger and a Devils hockey player are fighting on the ice. The Devils player ($M$ = 100 kg) throws a punch that sends the Ranger ($m$ = 80 kg) off at $v_R = -0.5\ \text{m/s}$.

What is the speed of the Devils player, $v_D$?

$$P = p_R + p_D$$

$$P_i = P_f$$

$$p_{R_i} = 0;\ p_{D_i} = 0 \ \Rightarrow P_i = 0$$

$$\Rightarrow P_f = 0 \ \Rightarrow p_{R_f} + p_{D_f} = 0$$

$v_{R_f}$


$v_{D_f}$

$$p_{R_f} = (80\ \text{kg})(-0.5\ \text{m/s});\ \ p_{D_f} = (100\ \text{kg})v_{D_f}$$

$$\Rightarrow (-40\ \text{kg}\cdot\text{m/s}) + (100\ \text{kg})v_{D_f} = 0$$

$$\Rightarrow v_{D_f} = \frac{(40\ \text{kg}\cdot\text{m/s})}{(100\ \text{kg})} = 0.4\ \text{m/s}$$

# i-Clicker

Two boxes are tied together by a string and are sitting at rest in the middle of a large frictionless surface. Between the two boxes is a massless compressed spring. The string tying the two boxes together is cut suddenly and the spring expands, pushing the boxes apart. The box on the left has four times the mass of the box on the right.

At the instant (after the string is cut) that the boxes lose contact with the spring, the speed of the box on the left will be…


A.) Greater than the right box
B.) Less than the right box
C.) Equal to the right box
D.) Not enough information provided

# MOMENTUM CONSERVATION AND COLLISIONS

Collision: Brief, strong interaction between objects.

If $\vec{F}_{ext} << \vec{F}_i$ between objects, Neglect $\vec{F}_{ext}$

→ behaves as an isolated system

$$\vec{P}_F = \Sigma_i \vec{p}_{i_F} \quad = \quad \vec{P}_I = \Sigma_i \vec{p}_{i_I}$$

Total momentum just after collision

= Total momentum just before collision

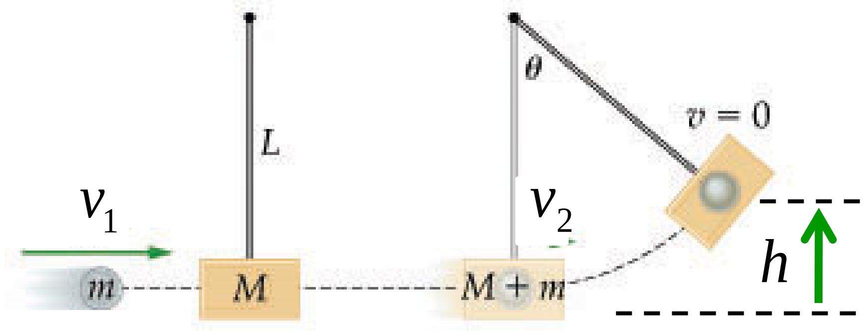
- Classify Collisions:
  - Elastic Collision — Total momentum and Total kinetic energy conserved
  - Inelastic Collision — Momentum conserved, KE is not (lost to internal energy)
  - Completely Inelastic Collision (Objects stick together) — Momentum conserved, KE not. (KE → Internal)

Momentum conserved in any collision

KE conserved only in elastic collision

# EXAMPLE: Completely Inelastic Collision

The Ballistic Pendulum:




A bullet $(m_a, v_1)$ is fired into clip of pendulum which swings to height $h$. What is $v_1$ ?

TWO PARTS !

- Collision is completely inelastic

Use $\vec{P}_F = \vec{P}_I$ to find state just after collision.

$$\vec{P}_I = m_a v_1 + 0 \qquad P_F = (m_a + m_b) v_2$$

$$v_1 = \left( \frac{m_A + m_B}{m_A} \right) v_2$$

- Use conservation of mechanical energy:

$$K_2 + U_{g_2} = K_f + U_{gf}$$

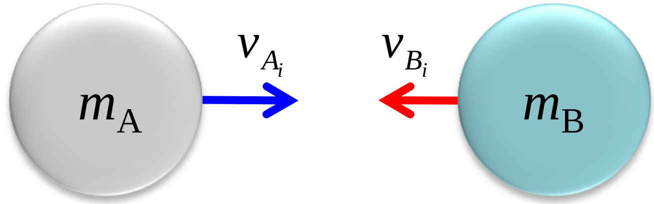
$$\tfrac{1}{2}(m_A + m_B) v_2^2 = (m_A + m_B) g h \;\Rightarrow v_2 = \sqrt{2gh}$$

$$v_1 = \left( \frac{m_A + m_B}{m_A} \right) \sqrt{2gh}$$

# ELASTIC COLLISION

- KE and $\vec{P}$ CONSERVED

"Billiard Ball Collision"




Before

1-D Collision
along $x$-axis
(omit subscripts)

After ?

1 $$\vec{P}_F = \vec{P}_I \Rightarrow m_A v_{A_2} + m_B v_{B_2} = m_A v_{A_1} + m_B v_{B_1}$$

2 $$KE_F = KE_I \Rightarrow \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2 = \tfrac{1}{2} m_A v_{A1}^2 + \tfrac{1}{2} m_B v_{B1}^2$$

⋮

PAGE OF ALGEBRA

⋮

3 $$(v_{B_2} - v_{A_2}) = -(v_{B_1} - v_{A_1})$$

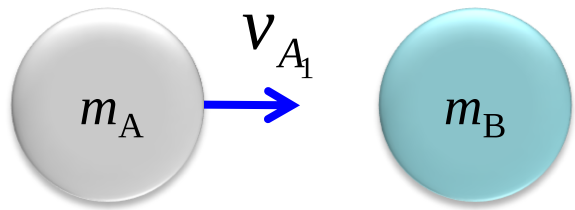
MAGNITUDE OF RELATIVE VELOCITY
UNCHANGED AFTER COLLISION

# EXAMPLE: Pocket The Eight Ball

**Before collision:**

$m_A$ moving

$m_B$ at rest




(1) $\Rightarrow m_A v_{A_1} + 0 = m_A v_{A_2} + m_B v_{B_2}$

(3) $\Rightarrow v_{A_1} - 0 = v_{B_2} - v_{A_2}$

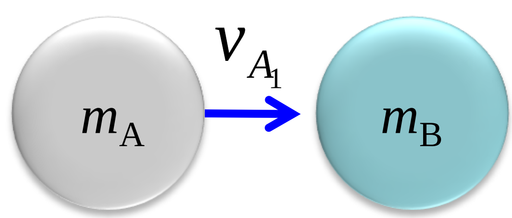
$$\Rightarrow v_{B_2} = \left(\frac{2m_A}{m_A + m_B}\right) v_{A_1}$$

$$v_{A_2} = \left(\frac{m_A - m_B}{m_A + m_B}\right) v_{A_1}$$

## 3 IMPORTANT CASES

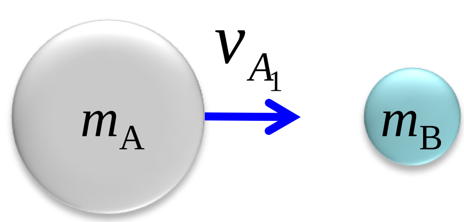
(I) $m_B = m_A$

$\Rightarrow v_{A_2} = 0 ; v_{B_2} = v_{A_1}$



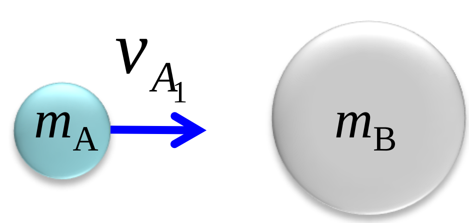

(II) $m_B << m_A$

$\Rightarrow v_{A_2} \cong v_{A_1}$ ; $v_{B_2} \cong 2v_{A_1}$



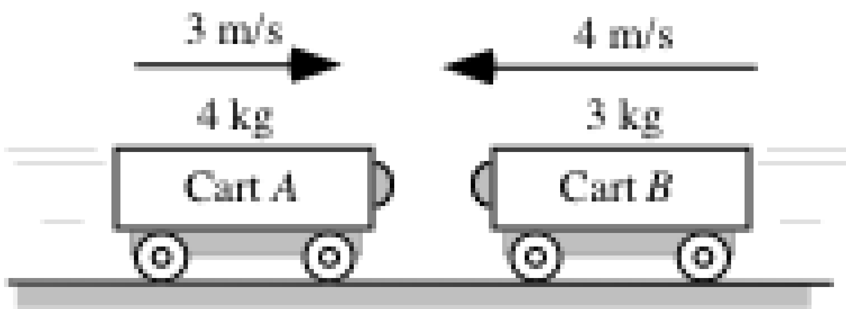

(III) $m_A << m_B$

$\Rightarrow v_{A_2} \cong -v_{A_1}$ ; $v_{B_2} \cong 0$



# i-Clicker

**Carts *A* and *B* are shown just before they collide. Which (if any) of the following statements could possibly be correct?**

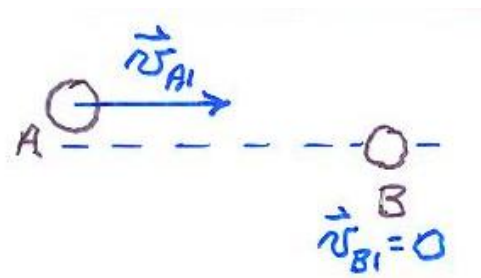



**I.** *"After the collision, the carts will stick together and move off to the left due to Cart B having more speed."*
***II.*** *"They'll stick together and move off to the right because Cart A is heavier."*
***III.*** *"The speed and the mass compensate. For completely inelastic collision, both carts are going to be at rest after the collision."*
***IV.*** *"For an elastic collision, they will change their directions, so Cart A will be moving to the left at 3 m/s and Cart B will be moving to the right at 4 m/s."*

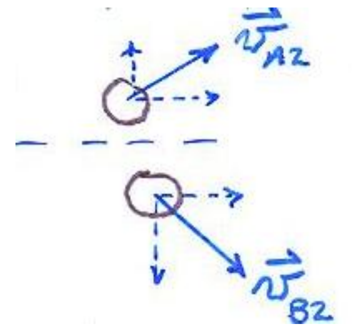
**A.)** **I** **B.)** **II**
**C.)** **III** **D.)** **IV**
**E.)** **III & IV**

<u>Collision in two dimensions</u> (horizontal plane)

Before:




After:




Write separate momentum conservation equations for components:

$$P_x: \quad m_A v_{Ax_1} + \cancelto{0}{m_B v_{Bx_1}} = m_A v_{Ax_2} + m_B v_{Bx_2}$$

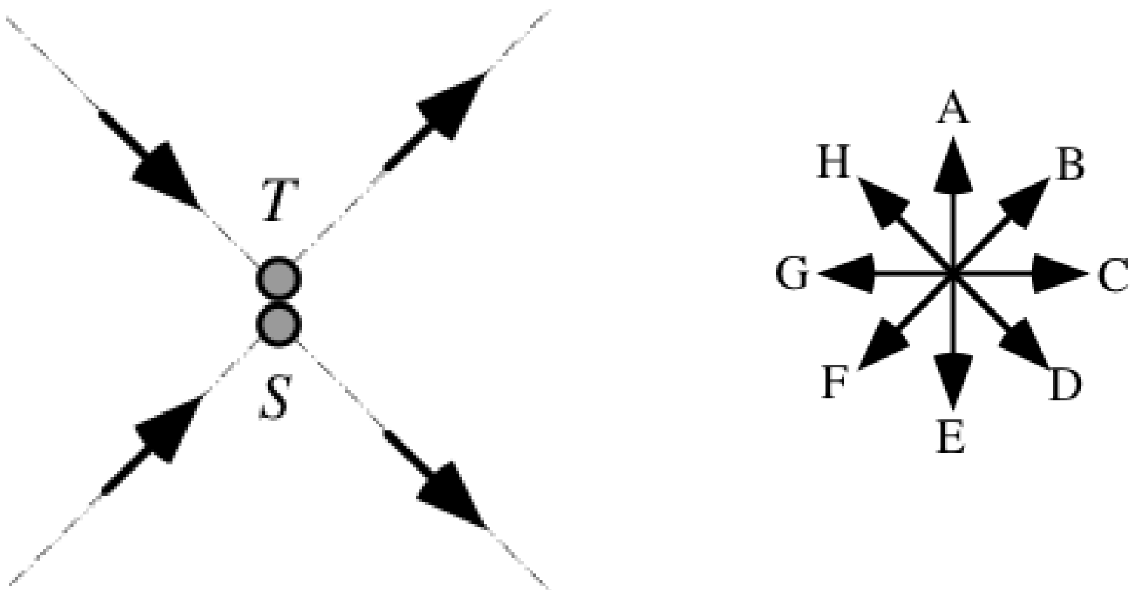
$$P_y: \quad \cancelto{0}{m_A v_{Ay_1}} + \cancelto{0}{m_B v_{By_1}} = m_A v_{Ay_2} + m_B v_{By_2}$$

If collision is elastic: $KE_1 = KE_2$

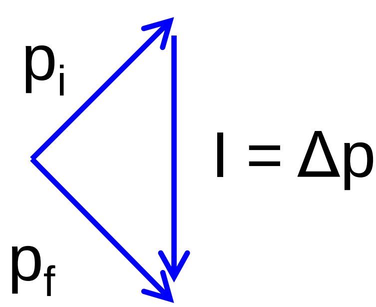
Three equations, can solve for a maximum of three unknowns:
Momentum and energy conservation alone are not sufficient to determine the final state.

# i-Clicker

Two identical steel balls, *S* and *T*, are shown at the instant that they collide. The paths and velocities of the two balls before and after the collision are indicated by the dashed lines and arrows. **What is the direction of the impulse on ball *S*?**

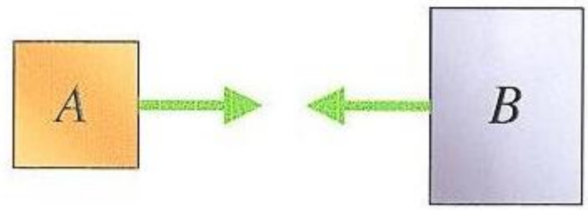



A. A
B. Cannot be determined without the time t.
C. C
D. None of the other answers.
E. E



# ***i-Clicker***

Two objects with different masses collide and *stick* to each other. Compared to *before* the collision, the system of two objects *after* the collision has




A. the same total momentum and the same total kinetic energy.

B. the same total momentum but less total kinetic energy.

C. less total momentum but the same total kinetic energy.

D. less total momentum and less total kinetic energy.

E. not enough information given to decide

- Completely Inelastic Collision Momentum conserved
  (Objects stick together) KE not. (KE → Internal)