Radiative Corrections and Two-Photon Exchange for MUSE Experiment

Andrei Afanasev

The George Washington University, Washington, DC, USA

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Objectives

- Use the best available practices to implement electromagnetic radiative corrections for MUSE
- Evaluate systematic uncertainties due to radiative corrections

Plan of talk

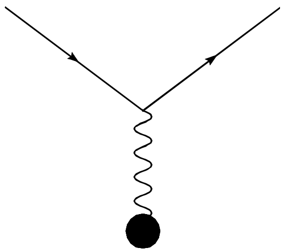
Radiative corrections for elastic charged lepton scattering

- Model-independent and model-dependent; soft and hard photons

Two-photon exchange effects

- Soft-photon exchange approximation and IR regularization
- Novel effects in muon scattering

Elastic Nucleon Form Factors



- Based on one-photon exchange approximation

$$M_{fi} = M_{fi}^{1\gamma}$$

$$M_{fi}^{1\gamma} = e^2 \bar{u}_e \gamma_\mu u_e \bar{u}_p (F_1(t)\gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)) u_p$$

- Two techniques to measure

$$\sigma = \sigma_0 (G_M{}^2 \tau + \varepsilon \cdot G_E{}^2) \quad : Rosenbluth \quad technique$$

$$\frac{P_x}{P_z} = -\frac{A_x}{A_z} = -\frac{G_E \sqrt{\tau}\sqrt{2\varepsilon(1-\varepsilon)}}{G_M \ \tau\sqrt{1-\varepsilon^2}} \quad : Polarization \quad technique$$

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2$$

$$(P_y = 0)$$

Latter due to: Akhiezer, Rekalo; Arnold, Carlson, Gross

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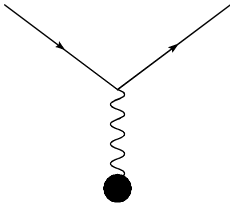
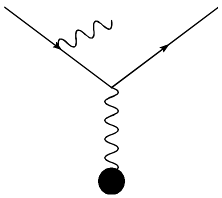
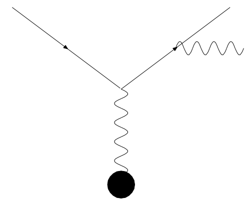
Basics of QED radiative corrections



(First) Born approximation

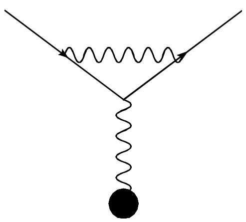


Initial-state radiation



Final-state radiation

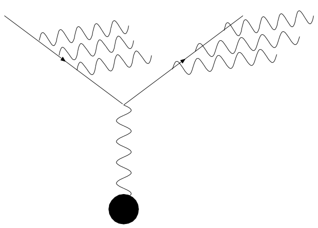
Cross section ~ dω/ω => integral diverges logarithmically: **IR catastrophe**



Vertex correction => cancels divergent terms; Schwinger (1949)

Assumed $Q^2/m_e^2 >> 1$

$$\sigma_{\exp} = (1+\delta)\sigma_{Born}, \quad \delta = \frac{-2\alpha}{\pi}\{(\ln\frac{E}{\Delta E} - \frac{13}{12})(\ln\frac{Q^2}{m_e^2} - 1) + \frac{17}{36} + \frac{1}{2}f(\theta)\}$$



Multiple soft-photon emission: solved by exponentiation, Yennie-Frautschi-Suura (YFS), 1961

$$(1+\delta) \rightarrow e^{\delta}$$

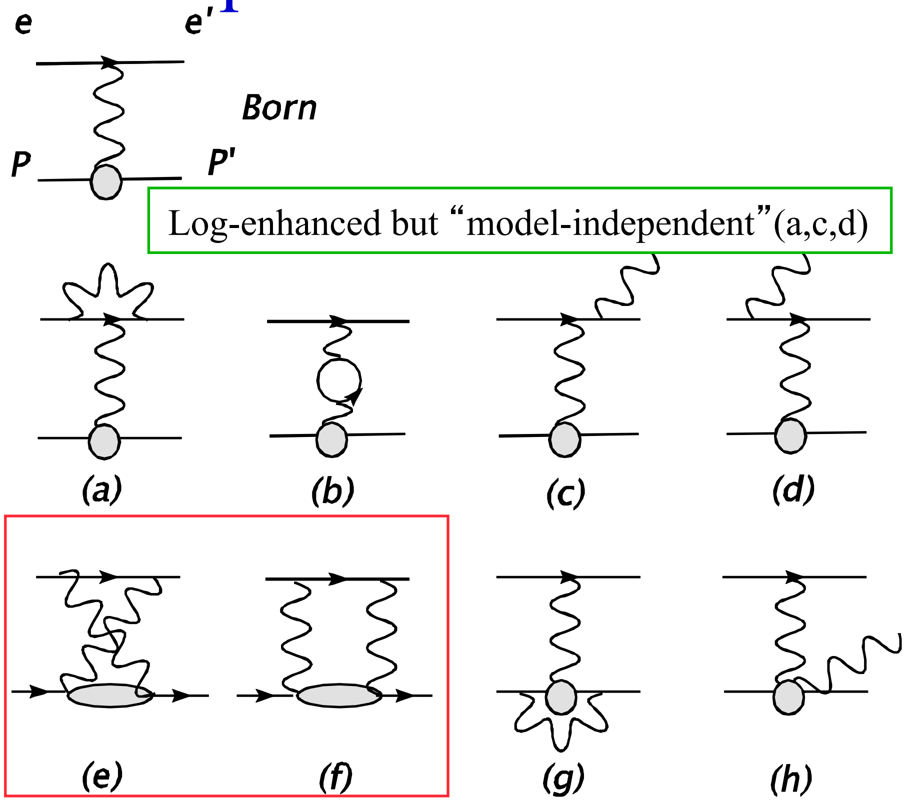
Approaches to QED Corrections for elastic ep

- L.W. Mo, Y.S. Tsai, Rev. Mod. Phys. 41, 205 (1969); Y.S. Tsai, Preprint SLAC-PUB-848 (1971).
  - Considered both elastic and inelastic inclusive cases. No polarization.
  - Updated my Maximon&Tjon, Phys.Rev. C62 (2000) 054320
- D.Yu. Bardin, N.M. Shumeiko, Nucl. Phys. B127, 242 (1977).
  - Covariant approach to the IR problem. Later extended to inclusive, semi-exclusive and exclusive reactions with polarization.
- Polarization case: Afanasev et al, PRD64 (2001)113009; PLB(2001)269
- A variety of Monte Carlo event generators are based on these basic approaches, e.g. ELRADGEN, AA et al., Czech.J.Phys. 53 (2003) B449.

Bremsstrahlung for Relativistic vs Nonrelativistic Lepton Scattering

- Accelerated charge always radiates, but the magnitude of the effect depends on kinematics
- See Bjorken&Drell (Vol.1, Ch.8):
  - For large $Q^2 >> m_e^2$ the rad.correction is enhanced by a large logarithm, $\log(Q^2/m_e^2) \sim 15$ for GeV^2 momentum transfers
  - For small $Q^2 << m_e^2$, rad.correction suppressed by Q^2/m_e^2
  - For intermediate $Q^2 \sim m_e^2$, neither enhancement nor suppression, rad correction of the order $2\alpha/\pi$

Complete radiative correction in O(α_{QED})



Radiative Corrections to elastic ep:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and Virtual Compton (g,h)
- Corrections (e-h) depend on the nucleon structure

Two-photon corrections: no large logs, but dependent on nucleon structure

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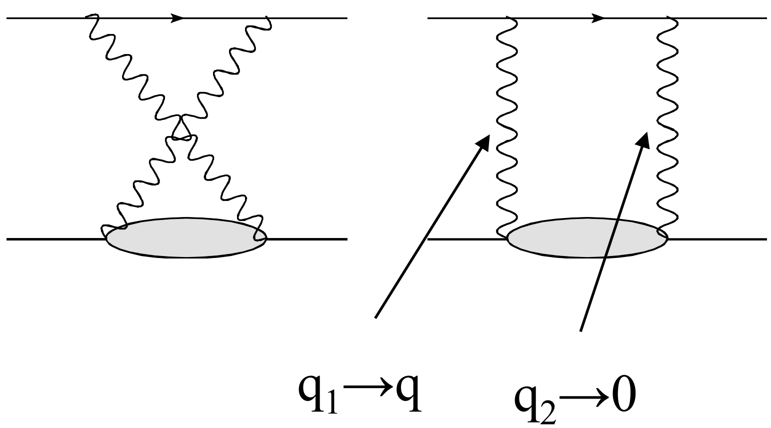
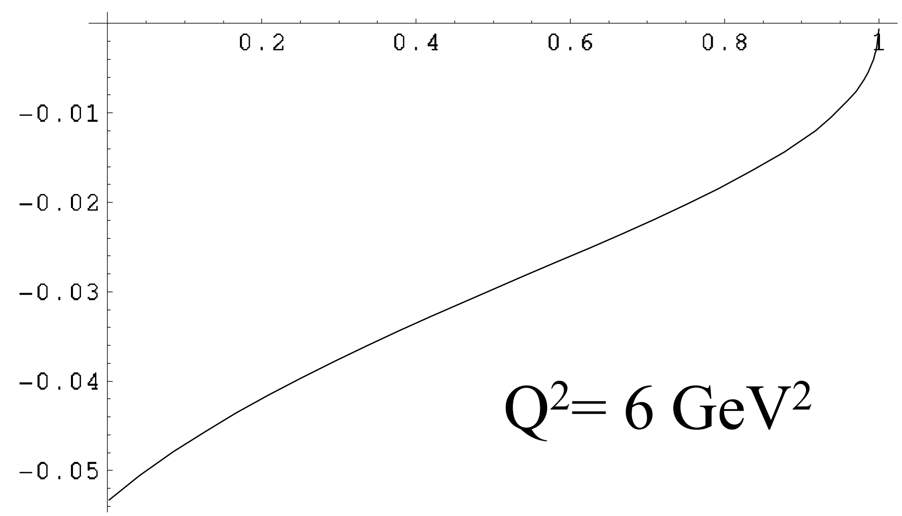
Separating *soft* 2-photon exchange

- Tsai; Maximon & Tjon ($k \rightarrow 0$); similar to Coulomb corrections at low Q^2
- Grammer &Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to cross section
- **Already included in experimental data analysis for elastic ep**
  - Also done for pion electroproduction in AA, Aleksejevs, Barkanova, arXiv:1207.1767 (there we also used Passarino-Veltman parameterization of loop integrals, inclusion of lepton masses is straightforward)





Lepton mass is not essential for TPE calculation in ultra-relativistic case;
Two-photon effect below 1% for lower energies and Q^2<0.1GeV^2

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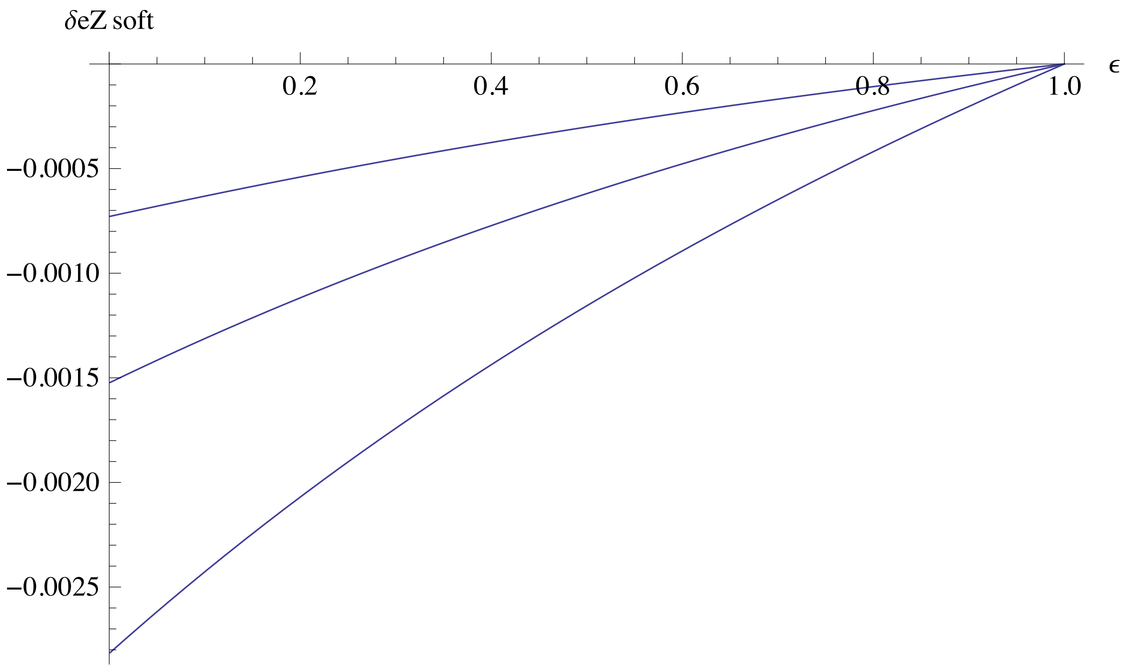
Soft TPE for low-energy electron scattering

- Soft TPE correction for ep scattering using Mo-Tsai formalism for three fixed electron energies: 115 MeV/c (top), 153 MeV/c (middle) 210 MeV/c (bottom curve); no large logs involved, muon correction is about the same: 0.25% max. Used kinematics for MUSE experiment proposal at PSI



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TPE for Proton Charge Radius Extraction

- Electron scattering: R_{ch} increase by +0.0015 fm, according to Blunden and Sick, Phys. Rev. C 72, 057601 (2005). *"The radius after two-photon correction amounts to 0.897+/-0.018 fm"*
- Recent calculations by Lee&Milstein, arXiv:1402.3054

Hard Bremsstrahlung

- Need to include radiative lepton tensor in a complete form:

AA et al, **Phys.Rev. D64 (2001) 113009; PLB 514, 269 (2001*):*** terms ~ k emitted photon momentum) usually neglected in rad.correction calculations, but can lead to ~1% effect for Rosenbluth slope at high Q^2

$$L^r_{\mu\nu} = -\frac{1}{2} Tr(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1+\gamma_5\hat{\xi}_e)(\hat{k}_1 + m)\overline{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\mu - \frac{\gamma_\mu \hat{k}\gamma_\alpha}{2k\cdot k_1} - \frac{\gamma_\alpha \hat{k}\gamma_\mu}{2k\cdot k_2}$$

$$\Gamma_{\alpha\nu} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\nu - \frac{\gamma_\alpha \hat{k}\gamma_\nu}{2k\cdot k_1} - \frac{\gamma_\nu \hat{k}\gamma_\alpha}{2k\cdot k_2}$$

additional terms, about 1% effect

common soft-photon approximation (Mo&Tsai;Maximon&Tjon)

Lepton Mass Effects

- Standard approximations keep the lepton mass in the logarithms but neglect it in power terms. May be justified in the ultrarelativistic case and $Q^2 \gg (\text{lepton mass})^2$
- Most of analysis codes use exact mass dependence for hard brem, but use above approximations for the "soft" part of brem correction
- Revised approach is required that will NOT result in new theoretical uncertainties
- New rad.correction codes no longer use peaking approximation (justified for relatively small lepton masses)
- Formalism and Monte-Carlo generators can be adapted for this analysis (ELRADGEN; MASCARAD, etc;

more on www.jlab.org/RC)

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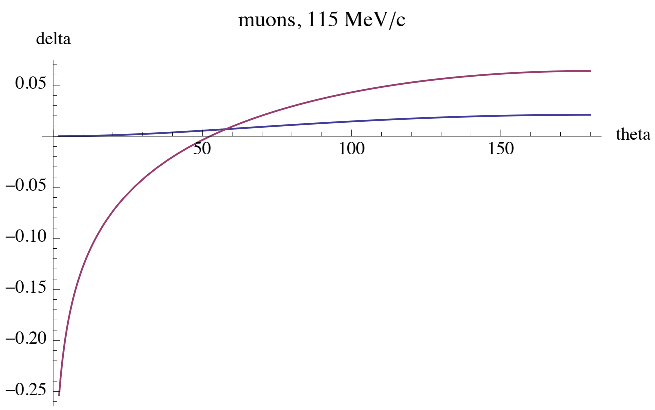
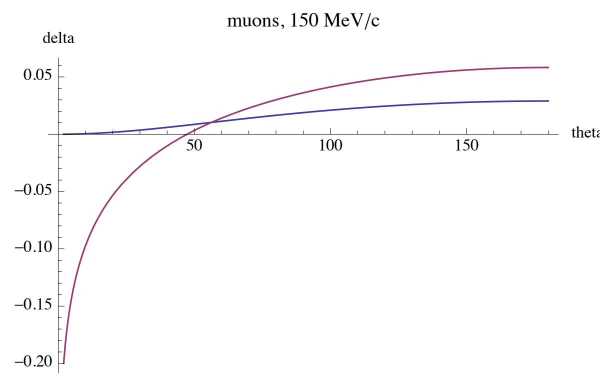
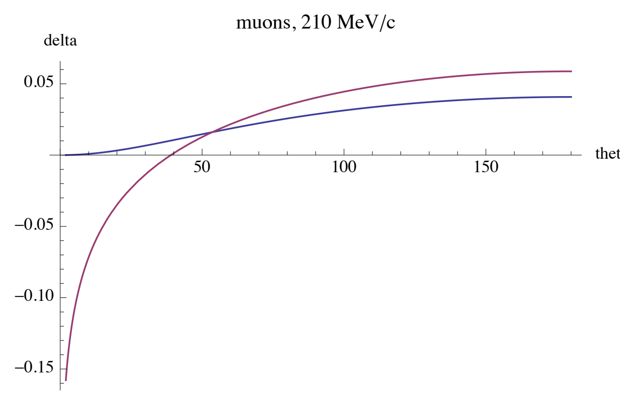
Brem corrections: Muons

- Mass effects for muons are essential; ultra-relativistic expressions (purple) no longer valid. Exact calcs (blue). Show brem correction vs lab scattering angle







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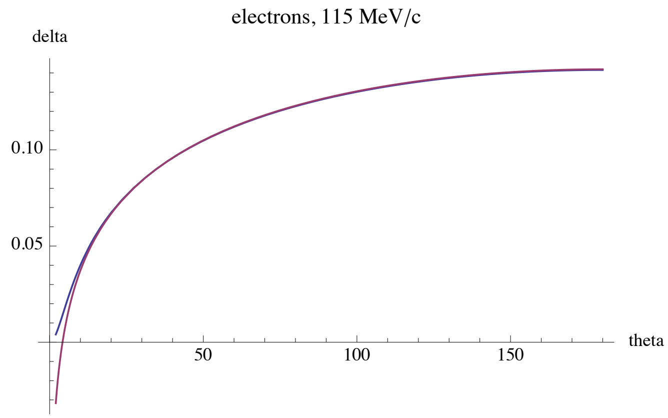
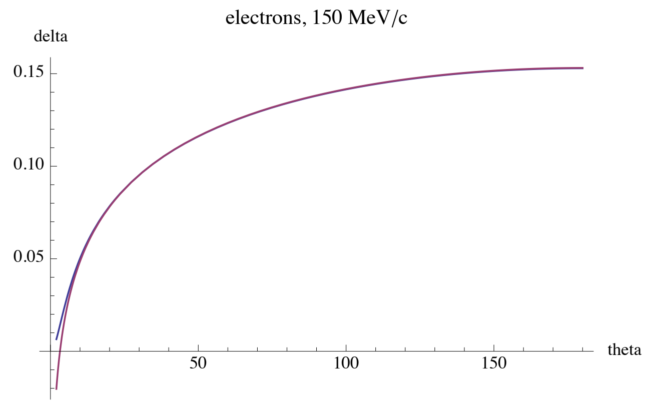
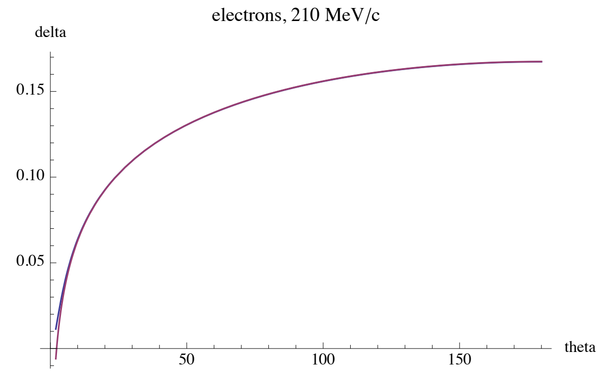
Brem corrections: Electrons

- Mass effects for electrons are less important; ultra-relativistic expressions (purple) give results close to exact calcs (blue). Standard rad.cor. codes are safe to use for MUSE.







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Coulomb and Two-Photon Corrections

- Coulomb correction calculations are well justified at lower energies and Q2
- Hard two-photon exchange (TPE) contributions cannot be calculated with the same level of precision as the other contributions.
- Two-photon exchange is independent on the lepton mass in an ultra-relativistic case.
- Issue: For energies ~ mass TPE amplitude is described by 6 independent generalized form factors; but experimental data on TPE are for ultrarelativistic electrons, hence independent info on 3 other form factors will be missing.
- Theoretical models show the trend that TPE has a smaller effect at lower Q^2 . The reason is that "hard" TPE amplitudes do not have a $1/Q^2$ Coulomb singularity, as opposed to the Born amplitude.

Helicity amplitudes for μp elastic scattering

- Total of 6 amplitudes:
  - 3 helicity-conserving, 3 helicity flip
  - Helicity-flip amplitudes neglected in ultra-relativistic $E_\mu \gg m_\mu$
  - Exception: single-spin beam asymmetries caused by interference of helicity-conserving and helicity-flip
- For muon scattering at ~100 MeV ultra-relativistic approximation no longer applies
- Model-independent analysis of two-photon exchange requires to fit amplitudes

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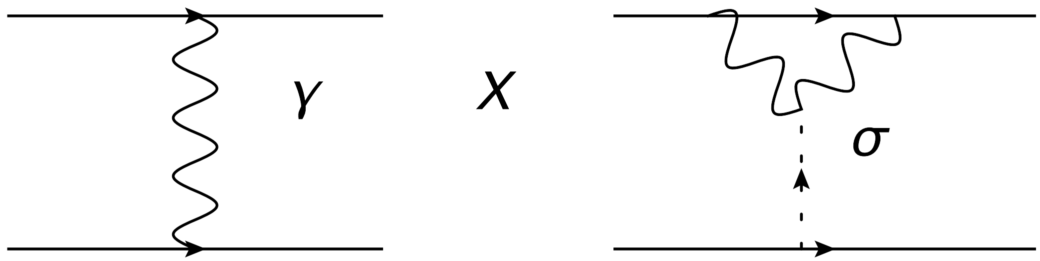
Helicity-Flip in TPE; estimate of inelastic contribution

- New dynamics from scalars (σ, f-mesons). No pseudo-scalar contribution for unpolarized particles
- Scalar t-channel exchange contributes to TPE (no longer setting m_{lepton} to zero!)



$$\delta_{\sigma}^{2\gamma} = -\alpha \frac{4\sqrt{\tau(1+\tau)(1-\varepsilon^2)}\, m_{\mu} M_N F_{\sigma\mu\mu} f_{\sigma NN} G_{Ep}}{(\tau G_{Mp}^2 + \varepsilon G_{Ep}^2)(Q^2 + m_{\sigma}^2)}$$

- No information on $F_{\sigma\mu\mu}$ is available. Need model estimates.

From sigma-pole contribution to nucleon polarizability, we estimate for Q^2=0.01 GeV^2 $\delta_{\sigma}^{2\gamma}$ is about 10^{-4}, lepton helicity-flip is important, scales as $\sqrt{\tau}$, $\tau = Q^2/4M_N^2$

(Special thanks to M. Pennington for input on σγγ coupling)

Can be studied directly in the ratio of μ+ and μ- cross sections

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Radiative corrections and systematics

- Bremsstrahlung corrections
  - Muons <3%
  - Electrons <15%

  Uncertainties determined by knowledge of experiment geometry, acceptances and kinematic cuts (See Steffen Strauch's slides)
- Two-photon exchange+brem intereference:
  - Electrons, muons <0.25%

  Can be measured/constrained by e^+/e^- and μ^+/μ^- comparison. Cancel in the sum of the cross sections.

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Summary

- Brem corrections are standard and well-established. Suppressed for muons; ultra-relativistic approximations no longer valid for muons, but can still be used for electrons in MUSE kinematics.
- Depend on detector acceptance, kinematic cuts and and analysis procedure =>Need to include into full Monte Carlo for MUSE (ELRADGEN, OLYMPUS generator and SIMC); see also S. Strauch's slides
- Two-photon exchange corrections are generally expected to be small, helicity-flip mechanism is enhanced for the muons
  - Can be studied directly in the ratio of μ+ and μ- cross sections
- We evaluate systematic uncertainty to cross sections from model-dependent radiative corrections at 0.1% for muon at Q^2~0.01 GeV^2, due to uncertainty in two-photon exchange effects

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