Ratios and Radius Extraction

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for the MUSE Collaboration

For determining if muons and electrons are the same:

1) Direct comparison of the scattering cross section and form factor
   - full Q^2 dependence
   - reduced systematics in form of ratio
2) Extracting the radius
   - low Q^2 behavior only
   - better sensitivity with relative comparison, not absolute

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Outline

- Systematics for the Cross Section
- Systematics for the Ratios
- TPE Expected Results
- μ/e Expected Results
- Extracting the Radius

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Relative Systematics Table

Solid Angle	0.1%
Scintillator Efficiency	0.1%
Beam Momentum Sensitivity	0.1%
Angle Determination	0.1%
Magnetic Contributions	0.1%
Multiple Scattering	0.3%
Radiative Corrections – μ	0.1%
Radiative Corrections – e	0.5%

Total Relative Uncertainty in Cross Section*:

μ: 0.4%

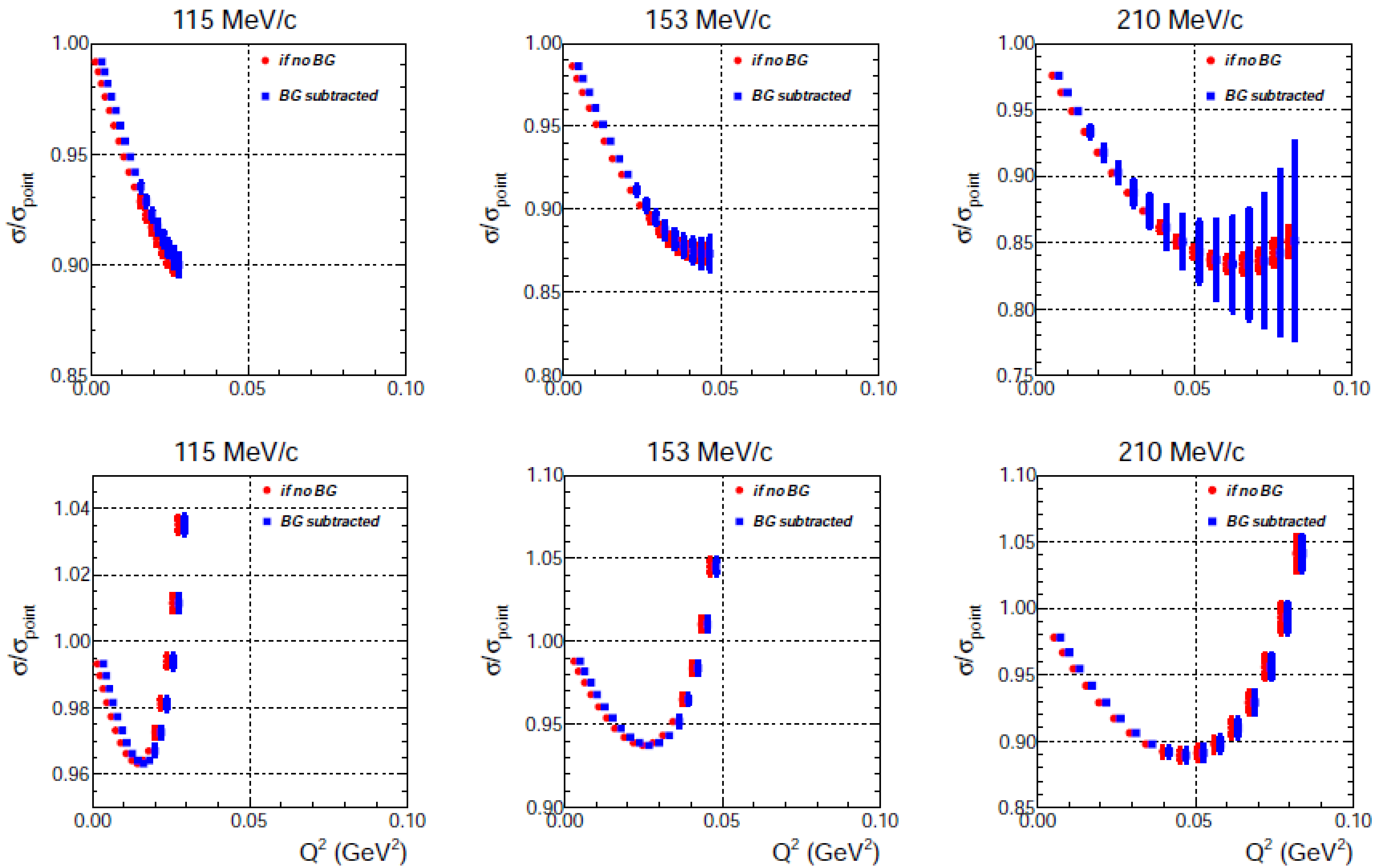
e: 0.6%

- Negligible Systematics:
  - Beamline Detector Efficiency
  - Beam Flux
  - Target Thickness
  - Data set Normalization
- TBD Systematics (small)
  - Analysis Uncertainties
  - Detector Stability

* Uncertainties factor of two smaller for form factor

Estimated Results

Cross Sections: μ^+p (top), e^+p (bottom) [Kelly FF's]



Offset in blue points for plotting

Statistical errors only

For electrons: Stat. errors well below 1%

For muons: Stat. errors below 1% for 115 and 153, above 1% at 210.

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Systematics for Ratios

- In the ratios (e+/e-, μ+/μ-, e/μ) some of the cross section systematics cancel further
- The uncertainty is reduced by a factor of 2 if we compare the form factor rather than the cross section:

  $d\sigma/d\Omega$ proportional to G^2
- Gain a normalization uncertainty of 0.2% (0.1%) for the cross section (form factor) ratios

TPE Ratios: **Syst. uncert: 0.3%**

- Comparing same particle, different polarity, same scattering angle
- Solid angle, angle determination uncertainties vanish
- Non-2 photon part of radiative correction vanishes
- Multiple scattering and magnetic contributions vanish

Systematics for Ratios

- In the ratios (e+/e-, μ+/μ-, e/μ) some of the cross section systematics cancel further
- The uncertainty is reduced by a factor of 2 if we compare the form factor rather than the cross section:

$$d\sigma/d\Omega \text{ proportional to } G^2$$

- Gain a normalization uncertainty of 0.2% (0.1%) for the cross section (form factor) ratios

Cross Section (FF) Ratios: **Syst. Uncert: 0.6% (0.3%)**

- Comparing different particle, slightly different scattering angle
- Majority of systematics remain
- Partial cancellation of scintillator efficiency, angle determination, multiple scattering, and magnetic contribution due to few-percent difference in angle for e, μ

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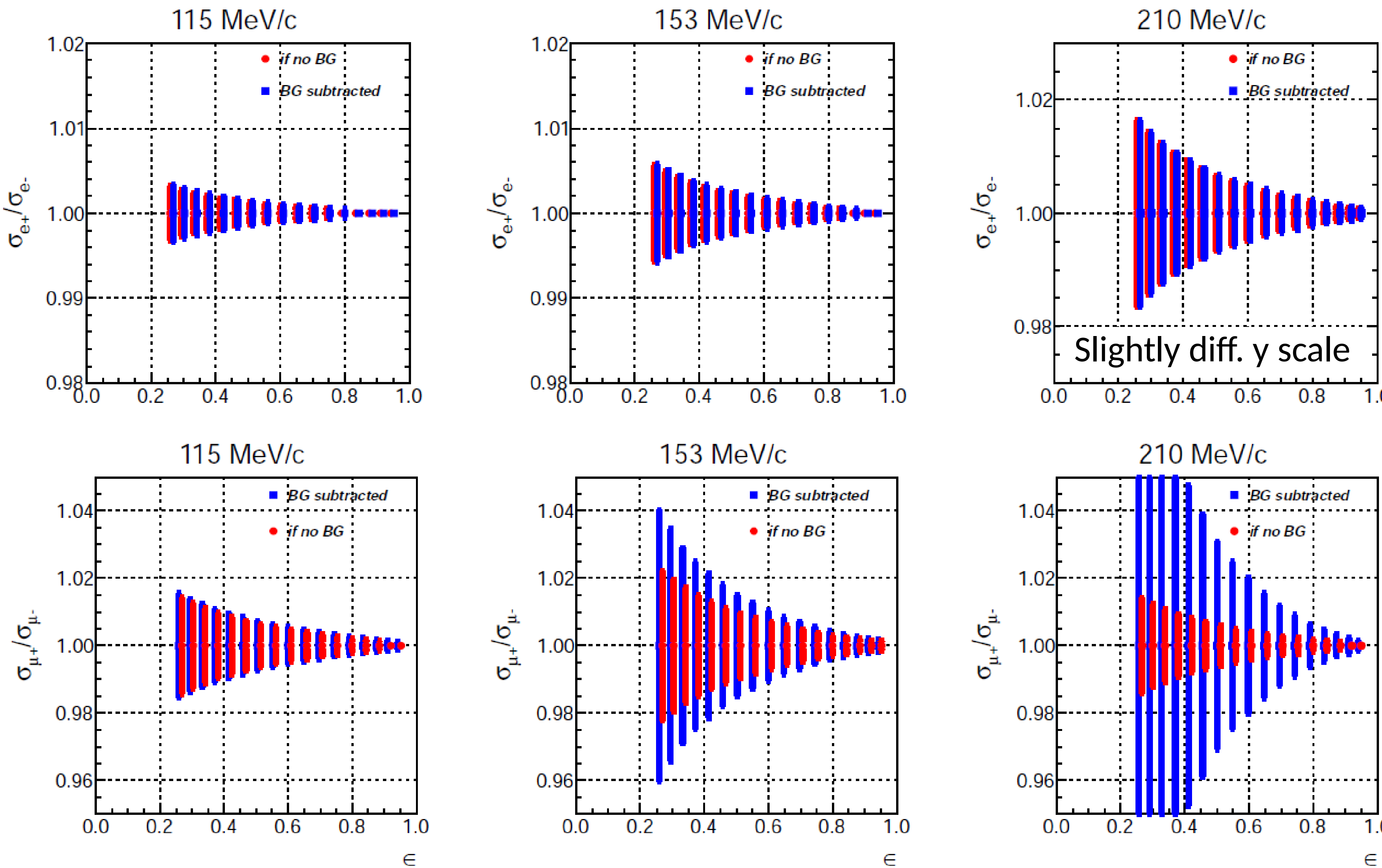
Estimated Results: TPE

Relative comparisons for e+/e- (top), mu+/mu- (bottom)



Stat. Errors only, but they dominate

Plotted versus epsilon -- virtual photon polarization

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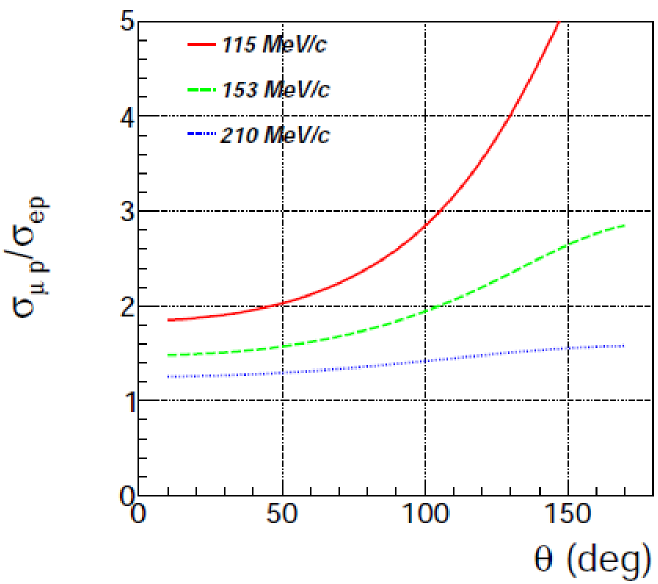
Estimated Results: μ/e

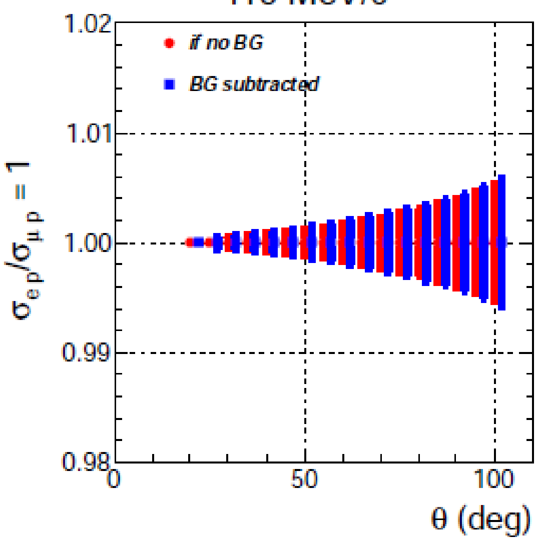


Left: Calculated difference in cross section
Below: Cross section ratio with statistical uncertainties

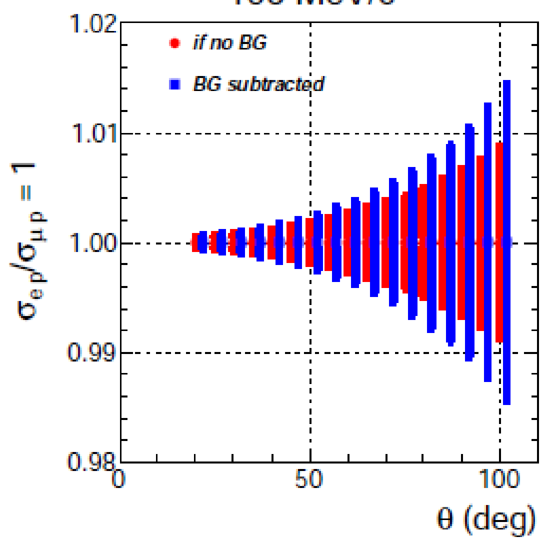
- Uncertainty reduced by factor of 2 in the form factor, leading to <1% statistical uncertainties for most of the dataset

(renormalized to unity, stat. errors only)

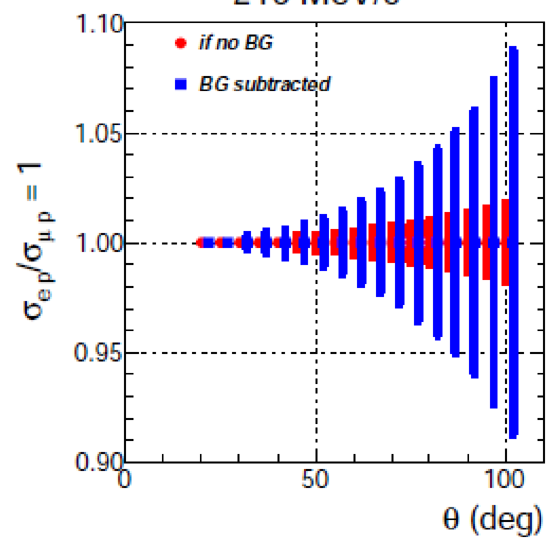
115 MeV/c



153 MeV/c



210 MeV/c



Slightly diff. y scale

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Extracting the Radius

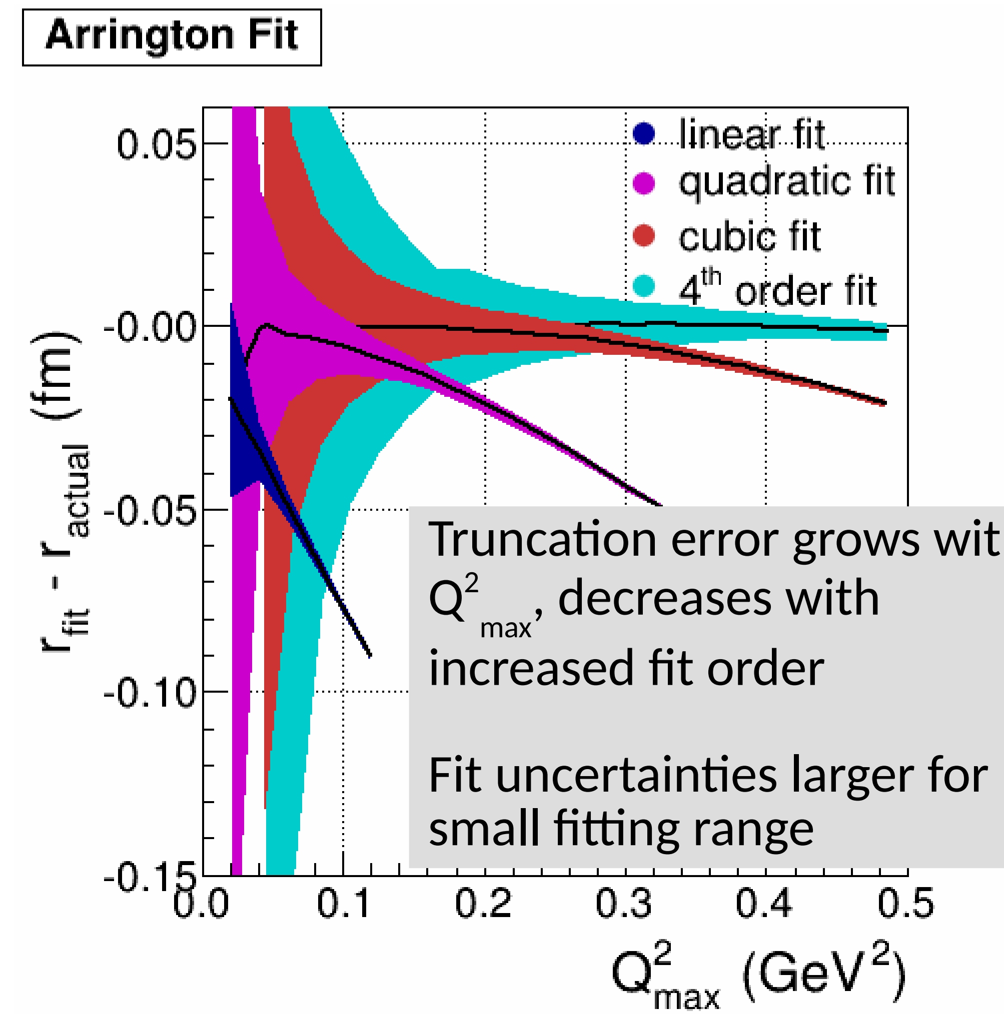
- Use truncated expansion and estimate the associated “truncation” error: the offset between the fitted radius and the input radius

Example to describe truncation error:

- Use Arrington form factor parameterization to generate pseudo-data from Q^2 = 0.004 GeV^2 to variable Q^2_{max} spaced every 0.001 GeV^2 with 0.4% point-to-point uncertainties

- Fit with truncated Taylor series of different orders

- Truncation error is difference between fitted and actual radius



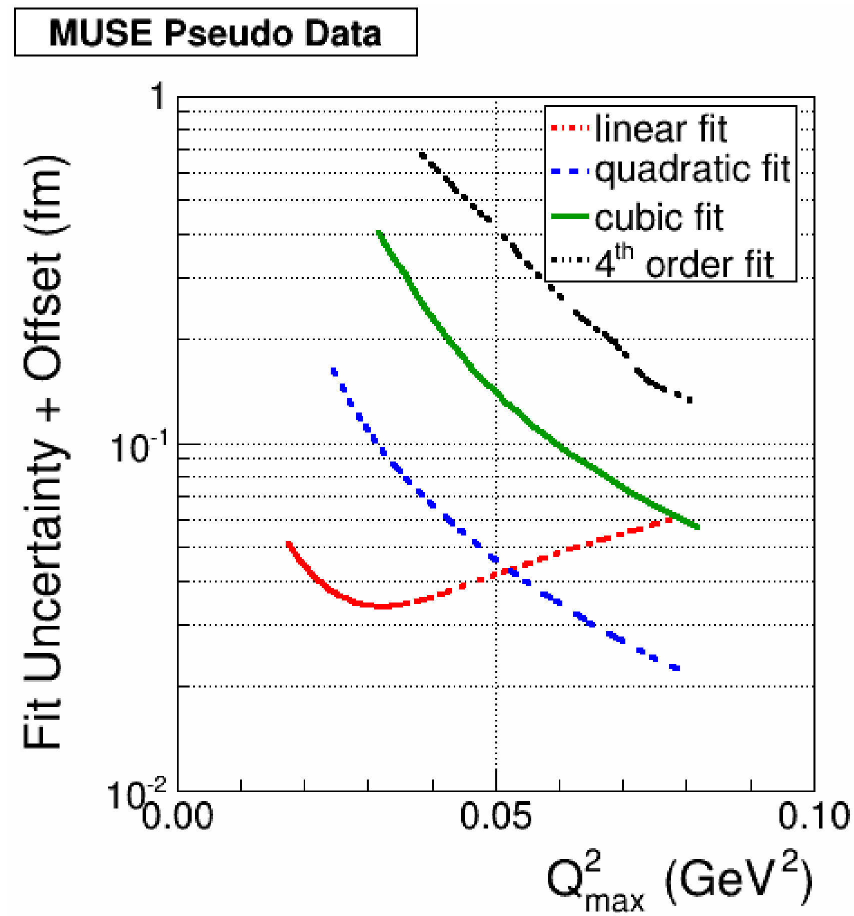
Truncation error grows with Q^2_{max}, decreases with increased fit order

Fit uncertainties larger for small fitting range

Extracting the Radius

- The low Q^2 region is applicable for our experiment
- How well the radius is determined is a combination of the truncation error and the fit uncertainties
  - Truncation error grows with the Q^2_{max} of fit
  - Fit uncertainties grow with reduced Q^2 fit range
- In example (right) with truncated Taylor series fit to all data:
  - Linear fit: 0.06 fm uncertainty (0.06 fm truncation error with 0.0053 fit uncertainty)
  - 2nd order fit: 0.02 fm uncertainty (0.003 fm truncation error with 0.02 fm fit uncertainty)

Fits to MUSE pseudo data
16 points from Q^2 = 0.0025 to 0.0775 GeV^2, 0.4% uncert.



Extracting the Radius

- We can use fit with demonstrated good analytic properties: inverse polynomial, continued fraction expansion, z-expansion
- Two independent extractions as example method (using single-parameter polynomial, so a conservative estimate):

  1) Use lowest energy setting with the smallest statistical uncertainties

     - Smaller truncation error, larger fit uncertainty, but overall dominated by experiment systematics: 0.017 fm

  2) With the two higher energy settings

     - Better fit but larger truncation error, which dominates the uncertainty: 0.016 fm

- Two methods with different systematics – combine to get an <u>absolute</u> error on the radius of 0.0120 fm (μ) and 0.0115 fm (e)

Extracting the Radius

- Can combine positive and negative polarity for improved statistics, but systematics unchanged
- For a relative e/μ comparison, uncertainties drop by factor of about 2
  - If e and μ have the same form factor – the truncation error will be the same
  - In practice, the truncation error may be slightly different (from differences in the form factor or from differences in data sets), but this will still be much smaller than overall size of truncation error and so the truncation error can be ignored

Extracting the Radius

Relative Radius Uncertainties

Combine polarity, relative comparison

δr = 0.007 fm (μ)

δr = 0.006 fm (e)

δr = 0.009 fm (μ-e)

Current discrepancy ~0.035 fm -> ~4σ measurement