Ph 442 Solutions for Project 1

1. a) Moving the cursor by one pixel vertically in the image of the Crab Nebula changes the declination by 4 arcseconds. Since the original Chandra ACIS detector pixels are 0.5 arcseconds across, the image has been binned 8×8.

b) I used a circular aperture centered at $(4110, 4053)$ with a radius of 142.6 pixels. This contained most of the flux from the nebula without extending beyond what looked like the edge of the detector. The results of fitting the different models to the spectrum from this region are given in the table below. There were 663 data points and 660 degrees of freedom.

Table 1: Fitted Spectral Parameters for the Entire Crab Nebula

Model	Flux observed (erg cm^{-2} s^{-1})	Flux corrected (erg cm^{-2} s^{-1})	N_H (10^{22} atoms cm^{-2})	kT (keV)	index	χ^2	χ^2/dof
powerlaw	1.69e-08	2.79e-08	0.343		1.95	55097	83.4804
blackbody	1.14e-08	1.17e-08	0.033	0.666		600908	910.467
bremsstrahlung	1.53e-08	2.12e-08	0.264	4.82		90828	137.618
mekal	1.70e-08	2.17e-08	0.235	5.70		282680	428.302

None of the models is a good fit to the data, *i.e* none has $\chi^2/\text{dof} = 1.0$. However, the fits of the power-law and bremsstrahlung models look reasonable by eye. The large χ^2 is due to the very high signal-to-noise ratio of the data and, probably, to a less-than-perfect model for the energy response function of the telescope and detector. The best-fitting spectral model is a power-law, suggesting that the emission is coming from synchrotron radiation.

c) The luminosity in the Chandra X-ray band implied by the flux from the power-law model is, for a distance of 2.0 kpc,

$$L_x = (2.79 \times 10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}) 4\pi (6.2 \times 10^{21} \text{ cm})^2 = 1.3 \times 10^{37} \text{ erg s}^{-1} = 3.5 \times 10^3 \ L_\odot. \quad (1)$$

d) The energy being lost by the Crab nebular pulsar as it slows down is given by equation (5.15) of the text:

$$\frac{dE_{rot}}{dt} = I\omega\dot{\omega} = I\left(\frac{2\pi}{P}\right)\left(\frac{2\pi\dot{P}}{P^2}\right) = 4\pi^2 I\left(\frac{\dot{P}}{P^3}\right). \quad (2)$$

The above used the relation between the angular velocity and rotation period, $\omega = 2\pi/P$. Using $I = 0.33MR^2$ with $P = 0.033$ s and $\dot{P} = 4.2 \times 10^{-13}$ s/s yields

$$\frac{dE_{rot}}{dt} = (6.9 \times 10^{38} \text{ erg s}^{-1})\left(\frac{M}{1.35\ M_\odot}\right)\left(\frac{R}{13 \text{ km}}\right)^2 \quad (3)$$

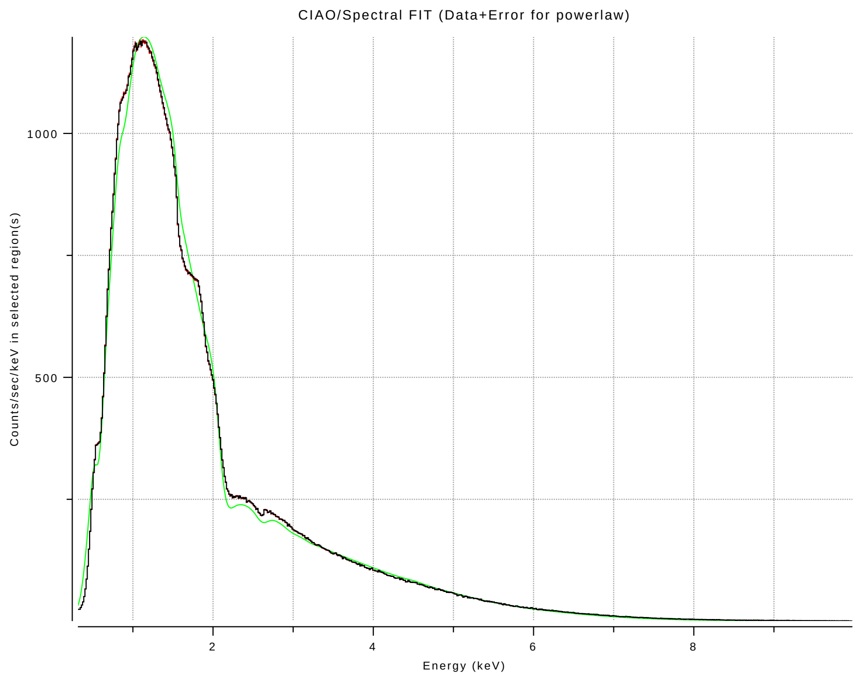

Figure 1: The spectrum of the Crab Nebula observed by Chandra and a fitted power-law model.

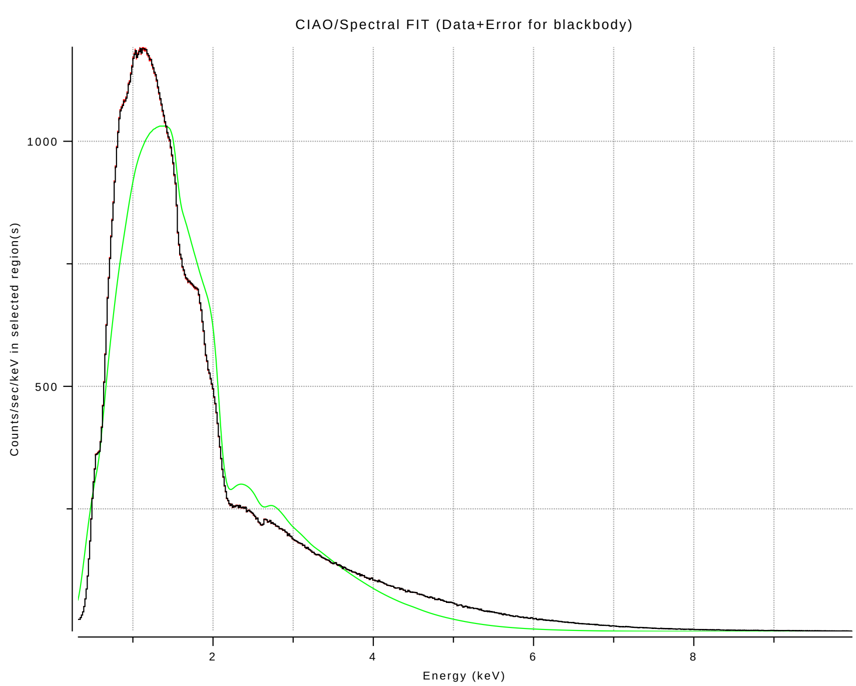

Figure 2: The spectrum of the Crab Nebula observed by Chandra and a fitted blackbody model.

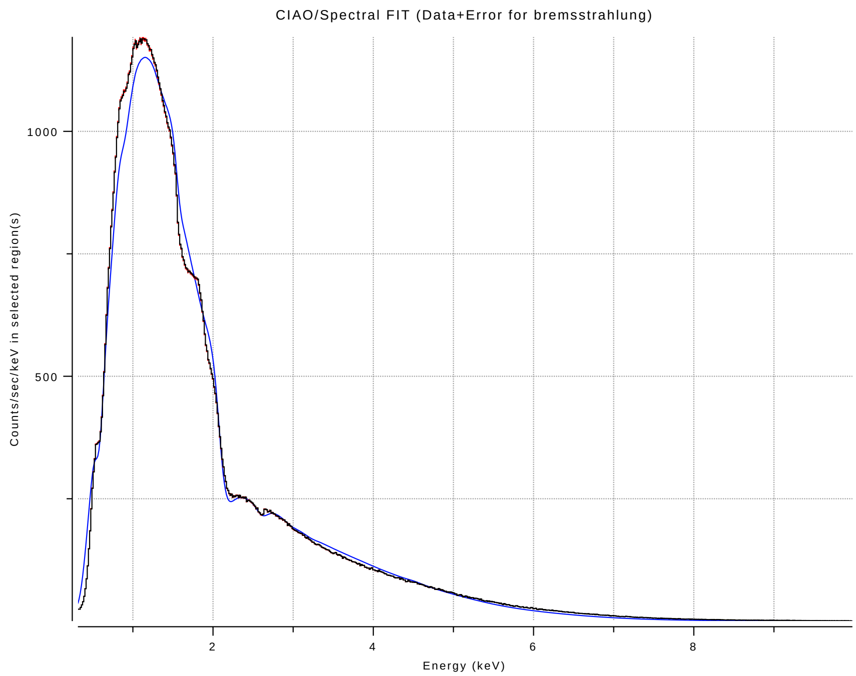

Figure 3: The spectrum of the Crab Nebula observed by Chandra and a fitted bremsstrahlung model.

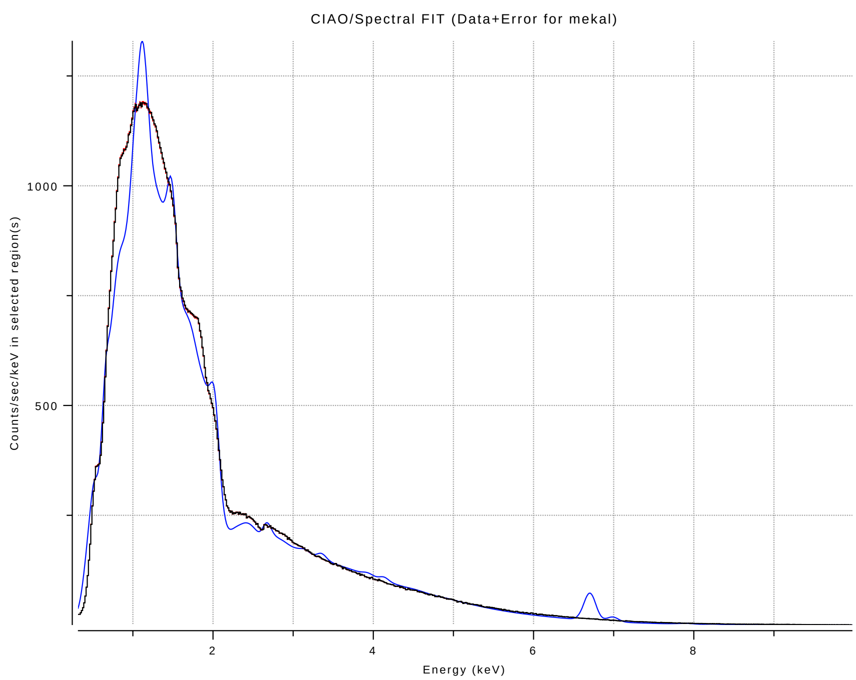

Figure 4: The spectrum of the Crab Nebula observed by Chandra and a fitted mekal model.

Thus, the slowing down of the Crab pulsar is easily able to power the x-ray luminosity detected by the Chandra Observatory for plausible values for the mass and radius of the neutron star in the pulsar.

e) The results of fitting a power-law model to the three regions of the nebula are given in the table below.

Table 2: Fitted Power-law Parameters for Regions of the Crab Nebula

Region	center (pixels)	radius (pixels)	N_H (10^{22} atoms cm^{-2})	s	χ^2/dof	p
center bright	$(4111, 4086)$	20.7	0.346	1.70	8.15	4.4
lower-left jet	$(4044, 3979)$	29.6	0.326	2.08	2.77	5.2
upper-right weak	$(4168, 4163)$	29.6	0.396	2.73	4.97	6.5

The fits all looked good in the plots. The fitted value of N_H does not vary significantly between the three regions. However, the spectral index, s, becomes larger (a spectrum that falls more steeply with increasing photon energy) as the regions move away from the center of the nebula.

f) The relation between the spectral index, s, of synchrotron radiation and the power-law index of the electron energy spectrum, p, is $p = 2s + 1$. The values of p for the three regions of the nebula are given in the final column of the table above.

The power-law index gets larger as the region moves from the bright center of the nebula to the outer part of the jet to the faint outer part of the nebula. The increasing index means that there are fewer high-energy electrons. A plausible explanation for this trend is that the high-energy electrons are losing their energy by synchrotron radiation as the plasma flows away from the pulsar at the center of the nebula. Since the synchrotron cooling time decreases as the electron energy increases, this energy loss causes the particle spectrum to steepen.

2. This problem had you examine the evidence for cosmic-ray acceleration in the strong shock marking the outer boundary of the remnant of SN 1006.

a) The relation between the observed power-law spectral index, s, and the electron energy-spectrum index, p, is $p = 2s + 1$ for synchrotron radiation. If $s = 0.6$, then $p = 2.2$. This value is close to the 2.0 predicted by the first-order Fermi mechanism.

b) The filaments in the image varied in width. I chose the width of the thinnest features based on the argument that these were the spots where we were seeing the shock exactly edge-on. Unfortunately, this image is binned 8×8 from the original Chandra pixels. The width of the thinnest filaments is about 1 binned pixel, which is 4 arcseconds. Some people were confused by measuring the difference in right ascension between two edges of a filament.

Right ascension is measured in units of time, where 1 second of time equals $15\cos(declination)$ seconds of arc. An angle of 4 arcseconds is 1.9×10^{-5} radian. For a distance to the remnant of 2.2 kpc, the implied physical width is 0.043 pc. This is an underestimate as careful measurements with the unbinned images that include the fainter brightness levels yield widths closer to 20 arcseconds.

c) I used a long, thin, crescent-shaped region containing the brightest filament near the top of the remnant. The region was about 50 binned pixels long and 6 binned pixels wide at its widest point. The center was approximately at the physical pixel $(4310, 4230)$. Fitting power-law, bremsstrahlung, and mekal models to the spectrum for this region yielded values for the χ^2 per degree of freedom of 1.22, 1.63, and 9.74, respectively. The power-law model is the best fit and has $N_H = 4.1 \times 10^{21}$ cm^{-2} and a spectral index of $s = 2.62$. The implied index for the power-law electron energy spectrum is then $p = 6.24$. These spectral and energy distribution-function indices are much larger than those found at radio wavelengths. This is interpreted as being due to the X-ray wavelengths being emitted by electrons with energies that are near the maximum generated by the first-order Fermi acceleration mechanism operating in the shock. We see a steeper decline of the number of electrons with increasing energy because the electron energy spectrum is cutting off at these energies.

d) The relation between the frequency of synchrotron photons typically emitted by an ultra-relativistic electron a relativistic factor γ and $\sin(\alpha) = \pi/4$ is (I used the expression for ω_c from my class notes, which is a factor of $1.5\times$ larger than equation (3.169) in the text — the definition of ω_c is slightly arbitrary):

$$\nu = 0.29\nu_c = 0.29\left(\frac{3}{4\pi}\right)\gamma^2\left(\frac{eB}{mc}\right)\sin(\alpha) \tag{4}$$
$$= (9.6 \text{ Hz})(B/10\ \mu\text{G})\gamma^2. \tag{5}$$

For this problem, it is more convenient to have an expression involving the energy of the photon $E_{ph} = h\nu$ and the energy of the electron, $E = \gamma mc^2$:

$$\left(\frac{E_\nu}{5 \text{ keV}}\right) = \left(\frac{h\nu}{5 \text{ keV}}\right) = (3.0 \times 10^{-3})\left(\frac{E}{10^{13} \text{ eV}}\right)^2\left(\frac{B}{10\ \mu\text{G}}\right) \tag{6}$$
$$\Rightarrow E = (1.8 \times 10^{14} \text{ eV})\left(\frac{E_\nu}{5 \text{ keV}}\right)^{1/2}\left(\frac{B}{10\ \mu\text{G}}\right)^{-1/2}. \tag{7}$$

For a magnetic field strength of $B = 40\ \mu$G, $E = 9.1 \times 10^{13}$ eV or 91 TeV.

e) The characteristic timescale for an electron with energy E to lose energy due to synchrotron radiation (the *synchrotron cooling time*) is given by equation (3.163) in the text

$$\tau = \frac{E}{(dW/dt)_{synch}} = (635 \text{ s})\left(\frac{B}{1 \text{ G}}\right)^{-2}\left(\frac{E}{1 \text{ erg}}\right)^{-1} \tag{8}$$
$$= (1.3 \times 10^4 \text{ yr})\left(\frac{B}{10\ \mu\text{G}}\right)^{-2}\left(\frac{E}{10^{13} \text{ eV}}\right)^{-1}. \tag{9}$$

Substituting in equation 7 for E, the lifetime of the electron producing photons with energy E_ν is

$$\tau = (7.2 \times 10^2 \text{ yrs}) \left(\frac{E_\nu}{5 \text{ keV}}\right)^{-1/2} \left(\frac{B}{10 \ \mu\text{G}}\right)^{-3/2} . \tag{10}$$

For $B = 40$ μG, $\tau = 90$ yrs. This is much shorter than the $\sim$1000 yr age of the supernova remnant. Such energetic electrons would not be present unless there was a continuing source of them (*i.e.*, the acceleration of particles in the shock).

f) If the high-energy electrons are produced by the shock wave at the outer edge of the supernova remnant, then the width of the filaments producing X-ray emission might be determined by the timescale of their energy loss. As the electrons are advected away from the back of the shock they will lose energy and their X-ray emission will decrease. A strong non-radiating shock is the appropriate regime for the shock speed of the remnant of SN 1006. Then, in the frame in which the shock is at rest, the gas flows into the shock at v_{sh} and flows away from behind the shock at $v_{sh}/4$. Some people used $3v_{sh}/4$ as the velocity at which the plasma moves away from the shock. However, this is the velocity of the plasma behind the shock in the frame in which the gas ahead of the shock is at rest and this velocity is in the same direction as the motion of the shock (the shock accelerates the gas). In this frame the shock is moving at v_{sh}, so the motion of the gas away from the shock is still $v_{sh}/4$, as it must be.

Thus, the width of the X-ray emitting region is expected to be about $\ell = \tau v_{sh}/4$. Plugging equation 10 into this expression yields:

$$\ell = (0.53 \text{ pc}) \left(\frac{v_{sh}}{2900 \text{ km s}^{-1}}\right) \left(\frac{E_\nu}{5 \text{ keV}}\right)^{-1/2} \left(\frac{B}{10 \ \mu\text{G}}\right)^{-3/2} . \tag{11}$$

If $B = 40$ μG, then $\ell = 0.067$ pc. This is larger than our observed width of 0.043 pc, though not by a lot. A magnetic field of 54 μG would make the calculated and observed widths equal. A larger observed width would produce agreement with the 40 μG field strength.

If the above model is the explanation for the width of the X-ray emitting rim of the SN 1006 remnant, then we should observe the energies of the more-energetic electrons decreasing with increasing distance behind the shock. This would probably show up as the synchrotron spectral index increasing with increasing distance. This has not been observed in the SN 1006 remnant, but has been observed for the Tycho supernova remnant (Cassam-Chenai et al. 2007, ApJ, 665, 315).

g) Calculate the minimum energy needed to produce the synchrotron emission requires knowing the volume of the emitting region. Assuming that the emission comes from a thin spherical shell of width $dr = 0.043$ pc and radius $r = 9.6$ pc (corresponding to an angular diameter of 30 arcmin and a distance of $d = 2.2$ kpc) yields a volume of $V = 4\pi r^2 dr = 50$ pc^3 $= 1.5 \times 10^{57}$ cm^3. The luminosity at a frequency of 1 GHz implied by a flux of 19 Jansky is $L_\nu = 4\pi d^2 f_\nu = 1.1 \times 10^{23}$ erg s^{-1} Hz^{-1}. Plugging the above values for V and L_ν and

$\nu = 10^9$ Hz into

$$W_{min} = (1.1 \times 10^6 \text{ erg})\eta^{4/7} \left(\frac{V}{1 \text{ cm}^3}\right)^{3/7} \left(\frac{\nu}{1 \text{ Hz}}\right)^{2/7} \left(\frac{L_\nu}{1 \text{ erg s}^{-1}\text{Hz}^{-1}}\right)^{4/7} \quad (12)$$

yields $W_{min} = (1.7 \times 10^{46} \text{ erg})\eta^{4/7}$. This energy is much less than the 10^{51} erg of kinetic energy typical for a supernova remnant, even if η is 100. So the blast wave of the supernova remnant does have enough energy to produce the magnetic field strength and the high-energy electrons that produce the synchrotron radiation.

h) The minimum-energy solution for the synchrotron emission has

$$\begin{aligned} W_{mag} &= \frac{3}{4} W_{particle} = \frac{3}{4}(\frac{4}{7} W_{min}) & (13) \\ \Rightarrow V\left(\frac{B^2}{8\pi}\right) &= \frac{3}{7}(1.7 \times 10^{46} \text{ erg})\eta^{4/7} \left(\frac{V}{1.5 \times 10^{57} \text{ cm}^3}\right)^{3/7} & (14) \\ \Rightarrow B &= (11 \ \mu\text{G})\eta^{2/7} \left(\frac{V}{1.5 \times 10^{57} \text{ cm}^3}\right)^{-2/7}. & (15) \end{aligned}$$

To get $B = 40\mu$G requires $\eta = 92$. This value of η is comparable to that for cosmic rays arriving at Earth. Whether such a value is reasonable for the vicinity of the particle acceleration is still debated by researchers. It is worth noting that there is no requirement that a system satisfy the approximate equipartition of particle and magnetic energy required for the minimum energy solution. Though the post-shock magnetic fields implied by the width of the synchrotron-emitting rims around supernova remnants argue that these system are not too far from equipartition.