Ph 441/541 Problem Set 9

Due: Friday, April 27, 2012

In this homework set you will examine the results of some "real" stellar evolution calculations. These results were generated by the EZ code accessed through the EZ-Web interface (http://www.astro.wisc.edu/∼townsend/static.php?ref=ez-web). See that web page for some background information and the format of the output files produced by the program. There are two such files. A "summary" file provides a line of summary information (such as $\mathcal{M}$, $\mathcal{R}$, and $\mathcal{L}$, central temperature, effective temperature, ...) for each timestep of the evolutionary calculation. Note that columns 4–9 are the logarithm of the quantity rather than the quantity itself. A "structure" file provides the detailed run of physical quantities with radius within the model at one timestep.

The evolutionary calculations start with the star on the zero-age main sequence (ZAMS) and proceeds until the specified stop time or until the calculation runs into conditions that the code can't handle (usually some late stage of the evolution). The code can handle the helium flash, though only in an approximate manner.

1. Some illustrative stellar models:
Download the summary files available on the class website (linked under this problem set) for the four models with $\mathcal{M} = 0.4$, 1.0, 5.0, and 20.0 $\mathcal{M}_\odot$ and metallicity $Z = 0.02$ (approximately solar metallicity): m0.4z0.020-summary.txt, m1.0z0.020-summary.txt, m5.0z0.020-summary.txt, and m20.0z0.020-summary.txt.

a. At the initial time (the ZAMS) for each of these four models, what is the luminosity, $\mathcal{L}$, and radius, $\mathcal{R}$? What is the surface temperature? Use the table handed out early in the class to estimate the spectral type of each model.

b. Calculate for the four ZAMS models the ratios $G\mathcal{M}\mu m_A/(kT_c\mathcal{R})$, $G\mathcal{M}^2/(P_c\mathcal{R}^4)$, and $\rho_c/\langle\rho\rangle$, where T_c, P_c, and ρ_c and the central temperature, pressure, and density, respectively, $\langle\rho\rangle$ is the mean density, and μ is the mean molecular weight. How much do these ratios change with $\mathcal{M}$?

c. What fractions of the total luminosity are provided by the various nuclear reactions (pp, CNO, etc.) for the four ZAMS models? What fraction of the luminosity is emitted in the form of neutrinos?

d. What is the main-sequence lifetime of these four models? In other words, at what time does the central hydrogen fraction go to zero? How much larger is the luminosity of the star at this time compared to the ZAMS? What is the fractional change in the radius? What is the surface temperature at this point?

e. Plot the tracks of the these four models in the H-R diagram ($\log(\mathcal{L})$ *vs.* T_{eff}; use the surface temperature for T_{eff}). Importing the summary files into an Excel spreadsheet is one way to do this, though use whatever method you prefer. Mark the locations on the tracks where hydrogen is exhausted at the center of the star, where (significant) He burning by the triple-alpha process begins, and where (significant) energy generation by the burning of still heavier elements ("metal burning" in the table) begins. Not all of the tracks will have all of these points.

2. Models with 1 $\mathcal{M}_\odot$:
Download the summary file for the model with 1 $\mathcal{M}_\odot$ and a metallicity of $Z = 0.0001$: m1.0z0.0001-summary.txt.

a. How does the ZAMS luminosity, radius, and surface temperature of this metal-poor model compare to those of the 1 $\mathcal{M}_\odot$ model with solar metallicity ($Z = 0.02$)? Qualitatively explain why these quantities differ between the models.

b. Plot the evolutionary tracks of the solar-metallicity and metal-poor models on the H-R diagram. Note the very different surface temperatures of the core-helium burning (horizontal branch) stars for the two metallicities. This is a general feature of the horizontal branch. What are the surface temperatures of the bluest points on the horizontal branch?

c. At about what age does the solar-metallicity model reach a luminosity of 1.0 $\mathcal{L}_\odot$ and a radius of 1.0 $\mathcal{R}_\odot$? How does this compare with the age of the solar system from radioisotope dating of meteorites of 4.57 Gyrs?

d. Download the structure file corresponding to solar-metallicity model at a time approximately corresponding to the present: m1.0z0.020-today.txt. Over what range of radii is this model convectively unstable? How does this compare with the radius of the base of the convection zone for the Sun determined by helioseismology of 0.713 $\mathcal{R}_\odot$? What is the radius within which 95% of the total luminosity is generated in this model?

e. Using the structure file from the previous part, calculate at each radial point with significant nuclear energy generation the ratio of the energy generation per unit mass to $\rho^2 X^2 T^4$. Plot this ratio *versus* T. How good is our approximate formula for p-p energy generation?

3. Stellar structure and evolution (Ph 541 students only):
Download the four structure files for the ZAMS models with approximately solar metallicity: m0.4z0.020-zams.txt, m1.0z0.020-zams.txt, m5.0z0.020-zams.txt, and m20.0z0.020-zams.txt.

a. What regions in these models are convectively unstable? Express both as a range of radii and a range of enclosed mass.

b. What is the fraction of the total radius that contains 95% of the total luminosity generated? In what fraction of the total mass is that 95% of the luminosity generated?

c. Download the file m1.0z0.020-rgb.txt, which gives the structure for a 1 $\mathcal{M}_\odot$ model at a point just below the maximum luminosity on the red giant branch (the "tip" of the giant branch), which is step number 457 in the corresponding summary file. For what range(s) of radius and enclosed mass is this model convectively unstable? Referring to the summary file, what is the mass and radius of the helium core for this model? How does the temperature vary with radius in the helium core? Why is the temperature not maximum at the center of the core?

d. And finally.... Examine the summary file for the 20 $\mathcal{M}_\odot$ model. At what time does the fraction of the luminosity emitted in the form of neutrinos exceed 50% of the total luminosity?