# Ph 441/541 Problem Set 8

Due: Friday, April 13, 2012

1. Clayton model:
Consider a star of mass $\mathcal{M}$ and radius $\mathcal{R}$ in which the pressure gradient is given by

$$\frac{dP}{dr} = -\frac{4\pi}{3} G\rho_c^2 r \exp(-r^2/a^2),$$

where $a$ is a length parameter and $\rho_c$ is the central density; see Phillips Eq. (5.24). Derive an expression for the gravitational potential energy $E_{GR}$ of the star by using the virial theorem (Phillips Eq. (1.7)). Show that, if the length parameter $a$ is small compared with the radius $\mathcal{R}$, the gravitational potential energy is approximately

$$E_{GR} \approx -\frac{1}{3}\frac{\mathcal{R}}{a}\frac{G\mathcal{M}^2}{\mathcal{R}}.$$

2. Homologous stellar models:
Consider a family of stars in which the opacity is dominated by Thomson scattering by electrons and in which nuclear energy is generated by the carbon-nitrogen cycle. This implies that the opacity is independent of the density and temperature (see Phillips Eq. (5.13)) and that the rate of nuclear energy production is proportional to $\rho^2 T^{18}$ (see Phillips Section 4.2). In analogy with Phillips Problem 5.2, find for this family of stars a relation between the luminosity and the mass. Find also the line on the Hertzsprung-Russell diagram describing the luminosity and effective surface temperature for these stars.

3. Polytropes (Ph 541 students only):
Use the virial theorem to show that for a polytrope of index $n$, the gravitational potential energy is

$$E_{GR} = -\left(\frac{3}{5-n}\right)\frac{G\mathcal{M}^2}{\mathcal{R}}.$$