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(a few minor comments inside).

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Physics 326 Section 3
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LAB 6: DAMPED SIMPLE HARMONIC MOTION

Introduction

The main objective of this lab is to understand how damping simple harmonic motion affects the relationship between force F, position x, velocity v, acceleration a, and period T of a mass-spring system. Harmonic motion is defined as any motion that can be described by sin and cos functions. Undamped harmonic motion occurs when there is no frictional force that interferes with the oscillatory motion of the system so that the amplitude of motion remains unchanged over time. When frictional forces are small and the damping coefficient γ is smaller than the angular frequency ω, motion is considered underdamped and the amplitude of oscillation decays exponentially.

Experimental Method

The experimental setup (Figure 1) consisted of a ring stand with a spring and mass holder attached to a force transducer. The spring-mass system hung directly above a motion detector on the ground. The force transducer and motion detector were connected to the LabPro interface that transmits the position, velocity, acceleration, and force data to a computer. The computer program Logger Pro recorded the data.

Figure 1:

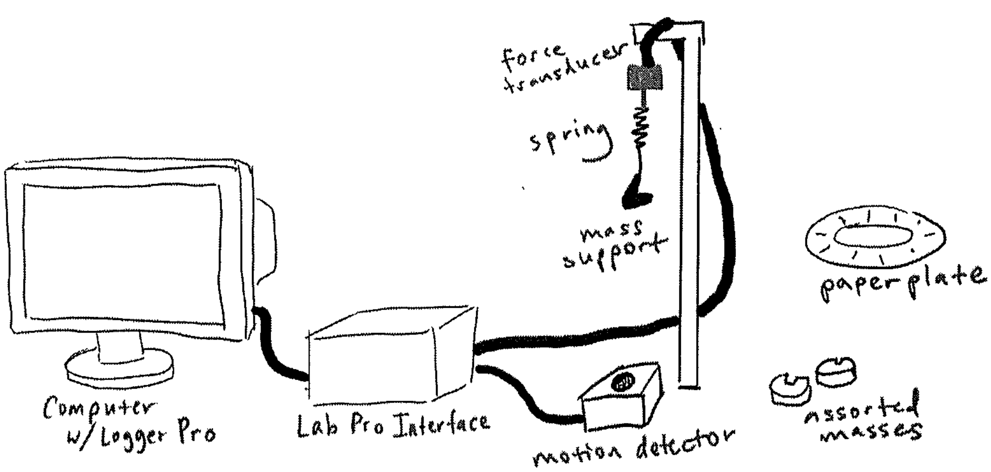

Figure 1: Diagram of experimental setup

We first aimed to determine the spring constant k using two methods. To do so, we assumed that the mass support was "massless", so that the spring with only the mass support attached is at equilibrium. For the first method, we measured the initial length of the spring in equilibrium position and recorded this value (Table 1). We then incrementally added 100g masses to the mass support, measuring the new length of the spring after the addition of each mass and recording it (Table 1). Using the formula $F = -kx$, we calculated the spring constant k for each mass. We averaged these values to get the mean spring constant.

We concurrently determined the spring constant k using the measurements given by the force transducer and motion detector. We first recorded the resting position of the mass support given via LoggerPro. After each weight was added to the mass support, we recorded the corresponding force and displacement (Table 2). Once again, using $F = -kx$, we calculated the spring constant k for each new mass and averaged the values to get the mean spring constant.

After configuring LoggerPro to record a 10 second run at a rate of 10 samples per second, we placed 500 g on the mass support. We pulled the mass down slightly and released it, allowing it to oscillate for a few periods before starting data collection. From the corresponding graphs of position versus time, we estimated the period T by locating various points where the curve passed through zero, measuring the distance between each of them, and taking the average of the measurements. We then calculated the relative phases of oscillation of velocity and acceleration relative to x.

Finally, to study damping effects in underdamped oscillation, we placed a paper plate under the weights. We configured the sampling rate to be 20 samples per second for a 15 second run and repeated the previous steps for data collection.

Results and Discussion

We calculated spring constant k to be [redacted] N/m by manually measuring the spring's extension x using a meter stick and solving for k using the formula $k = -\frac{F}{x}$. Since we added a 100 g mass after each measurement, we calculated the change in force between measurements to be $F = ma = (0.1\ kg) * \left(9.8\frac{m}{s}\right) = 0.98\ N$. Our measurements are shown below in Table 1:

Table 1:

Mass added (g)	**Length (m)**	**Displacement *x* (m)**	**Change in Force F (N)**	**Spring constant k (N/m)**
0 (equilibrium position)	[redacted]	N/A	N/A	N/A
100	[redacted]	[redacted]	[redacted]	[redacted]
200	[redacted]	[redacted]	[redacted]	[redacted]

300	[redacted]	[redacted]	[redacted]	[redacted]
Average spring constant $\bar{k}$				[redacted]

Table 1: Measurements of spring extension.

Since the error in the force measurement δF is 0.049 N/m (acceleration a is a constant and δm = 1 g = 0.005 kg) and the error in the extension measurement δx is 0.5 cm = 0.005 m, the error in the spring constant calculation δk is:

$$\delta k = k * \sqrt{\left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta x}{x}\right)^2} =$$ [redacted] $/m$

Now, using LoggerPro data, we recalculated the spring constant to be 21.6±0.2 N/m. Our measurements are shown in Table 2 below.

Table 2:

Mass added (g)	**Distance from motion sensor (m)**	**Displacement x (m)**	**Force (N)**	**Change in Force F (N)**	**Spring constant k (N/m)**
0 (equilibrium position)	[redacted]	N/A	[redacted]	N/A	N/A
100	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
200	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
300	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
Average spring constant $\bar{k}$					[redacted]

Table 2: Measurements of spring extension and force using the force and motion sensors.

Since the error in the force measurement δF is 0.0005 N/m and the error in the extension measurement δx is 0.0005 N/m, the error in the spring constant calculation δk is:

$$\delta k = k * \sqrt{\left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta x}{x}\right)^2} =$$ [redacted] N/m

k is more precisely measured using the LoggerPro measurements than the manual measurements.

QUESTION 1: HOW ACCURATELY DO YOU KNOW k?

We calculated our error in the measurement k to [redacted] for the method where we measured displacement with a meter stick and [redacted] for the method where we used LoggerPro data for the force and displacement measurements.

Figure 2:

Undamped?

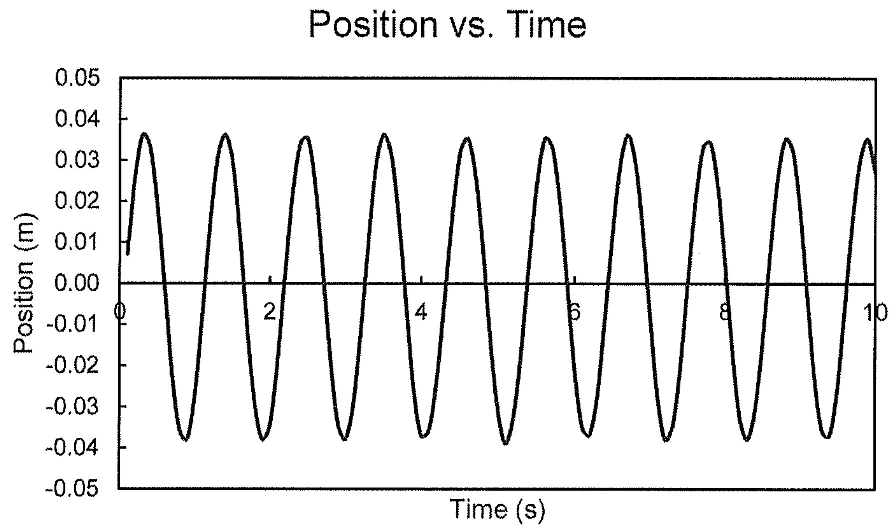

Figure 2: Position vs time for a 500 g oscillating mass. — indicate if this case is damped or undamped.

Figure 3:

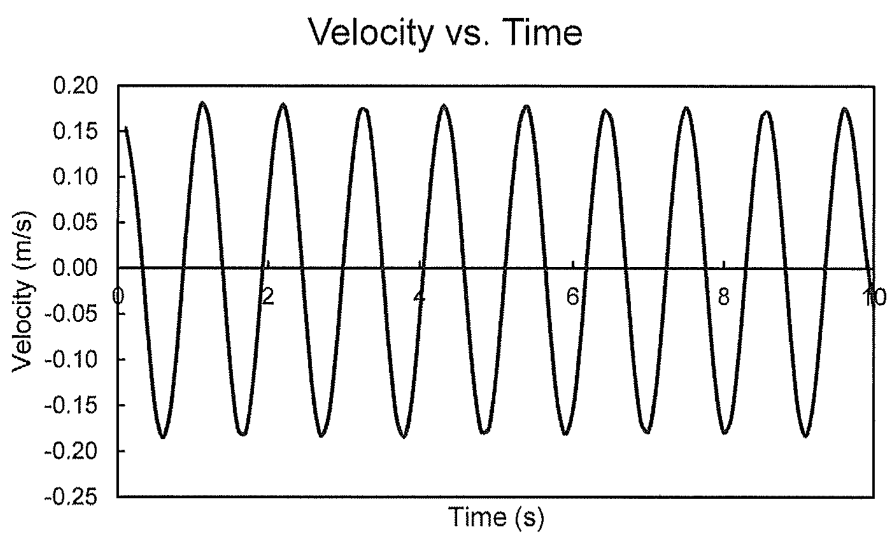

Figure 3: Velocity vs time for a 500 g oscillating mass.

Figure 4:

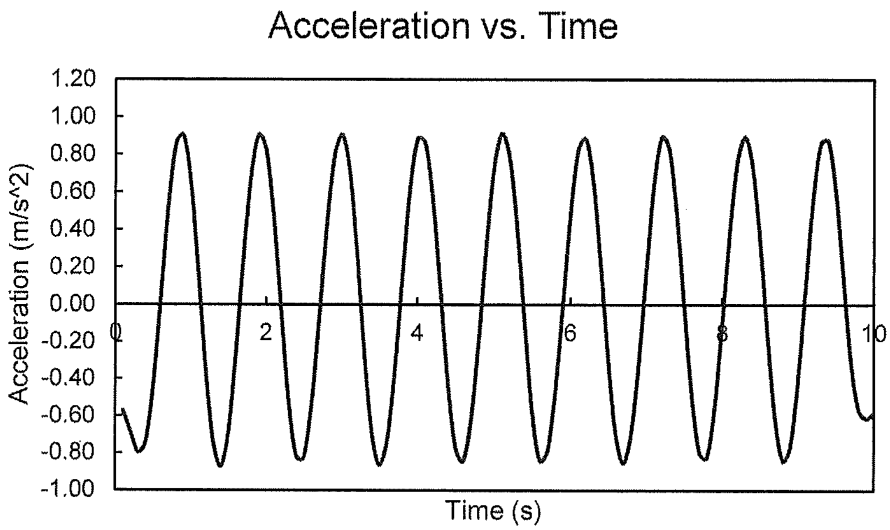

Figure 4: Acceleration vs time for a 500 g oscillating mass.

From Figure 2, we calculated the experimental period T to b[redacted]The measurements we used are shown in Table 3 below.

Table 3:

Time at peak (s)	**Time difference between peaks (s)**
[redacted]	N/A
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
Period (average peak to peak)	[redacted]

Table 3: Peak to peak measurements for calculating period.

QUESTION 2: SHOULD YOU INCLUDE THE MASS OF THE SPRING?

We do not worry about the mass of the spring, as the entire spring does not move with the system, is part of the system at rest, and has a negligible mass compared to the rest of the system's elements (masses and weight holder).

QUESTION 3: DO THE TWO CALCULATED VALUES OF *T* AGREE WITHIN THE ERROR LIMITS YOU HAVE DETERMINED?

We calculated the period *T* to [redacted] s off the graph of displacement vs. time (Figure 2). Since the force registered by the force [redacted]ducer while the mass holder was at rest was [redacted] N, we calculated the mass of the weight holder to be $m = \frac{F}{a} =$ [redacted] kg. Using the equation $T = 2\pi\sqrt{m/k}$, we calculated the theoretical period to [redacted] Our experimental period (from the graph) has [redacted] difference from the theoretical period (calculated above), so these two values agree [redacted] one another.

Next, we calculated the relative phases of oscillation using the formula $\Delta\phi = 2\pi * \frac{\Delta t}{T}$, where Δt is the time when the quantity in question (*x*, *v*, or *a*) first passes through 0. We [redacted]mined these values from the graphs of Figure 2, 3 and 4. We used the experimental period *T* [redacted] s.

Position:

$$\Delta\phi = 2\pi * \frac{\Delta t}{T} =$$ [redacted] radians [rad]

Velocity:

$$\Delta\phi = 2\pi * \frac{\Delta t}{T} =$$ [redacted] rad

Acceleration:

$$\Delta\phi = 2\pi * \frac{\Delta t}{T} =$$ [redacted] rad

Position and acceleration have the same phase shift; however, their graphs are perfectly out of phase (Figures 2 and 4). Since force is a function of mass and acceleration and the mass remains constant, I expect that position and force will also be entirely out of phase. This is evident on the plot of force vs position (Figure 5), as force is inversely related to position.

This means they have a phase shift of a $-\pi$, or a $+\pi$.

$$\Delta\varphi = \varphi_{position} - \varphi_{acceleration} = \pm\pi.$$

Figure 5:

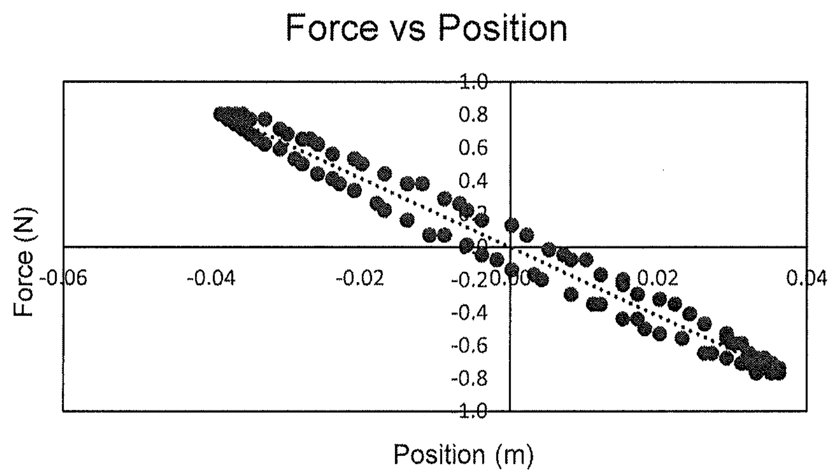

Figure 5: Force vs position graph.

Next, we constructed a phase space plot of velocity vs. position (Figure 6). The orbit is elliptical.

Figure 6:

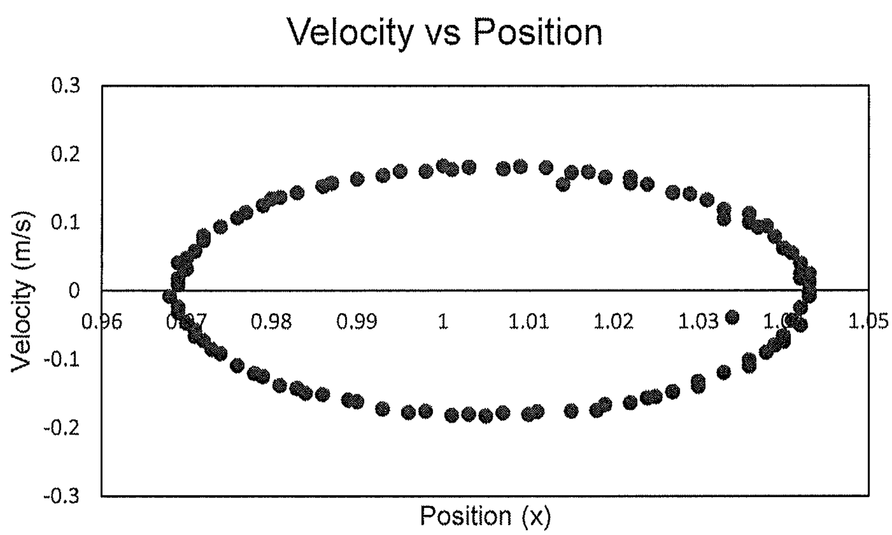

Figure 6: Velocity vs position phase space plot.

QUESTION 4: DERIVE A THEORETICAL EXPRESSION FOR THE SHAPE OF THIS ORBIT.

Using the formulas $x = A * \cos(\omega t + \varphi)$ and $v = A\omega * \cos\left(\omega t + \varphi + \frac{\pi}{2}\right)$ and knowing that our position and velocity at rest are x_0 m and v_0 m/s respectively, I calculated the theoretical expression for the shape of this orbit to be:

$$\frac{(x-x_0)^2}{(2A)^2}+\frac{(v-v_0)^2}{(2A\omega)^2}=1$$

QUESTION 5: WHAT IS THE ORBIT IN PHASE SPACE OF THE OSCILLATOR WHEN THE MASS IS AT REST.

Since the position of the mass is not changing and neither is its velocity, the phase space orbit would [redacted]

We then observed the effects of damping on oscillatory motion by placing a paper plate on the weight holder and recording another run. The corrected damping (adjusted to oscillate around the equilibrium position) is plotted below in Figure 7.

Figure 7:

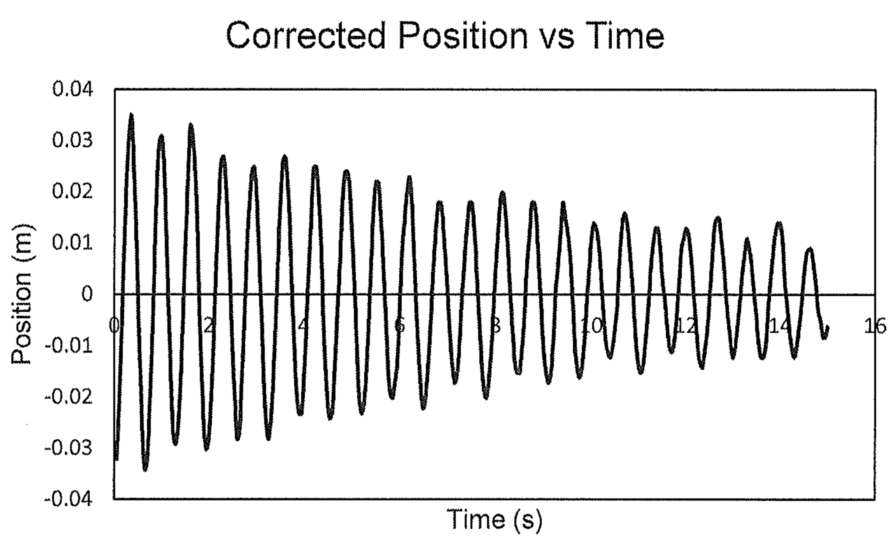

Figure 7: Position vs time for a damped 500 g oscillating mass.

The natural log of the amplitude vs. time is a straight line (Figure 8), as amplitude decays exponentially in underdamped oscillatory motion. We constructed a linear least square fit using initial max amplitude A_0=1.21 m to find a best-fitting val[redacted] for the damping constant γ (Figure 9). Air resistance is turbulent as opposed to viscous, since air has a very low viscosity but can move fast.

Figure 8:

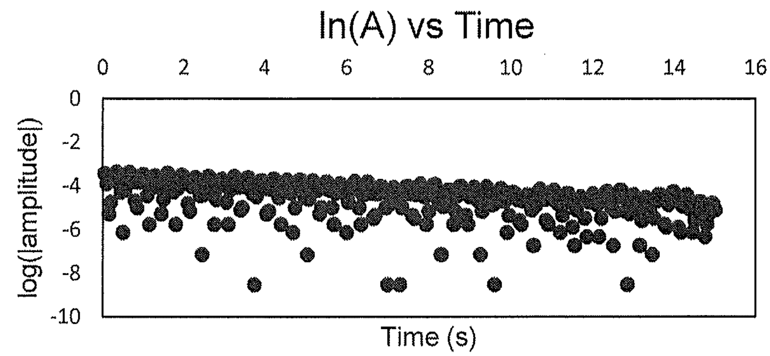

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Figure 8: Natural log of the absolute value of position vs. time for a damped 500 g oscillating mass.

Figure 9:

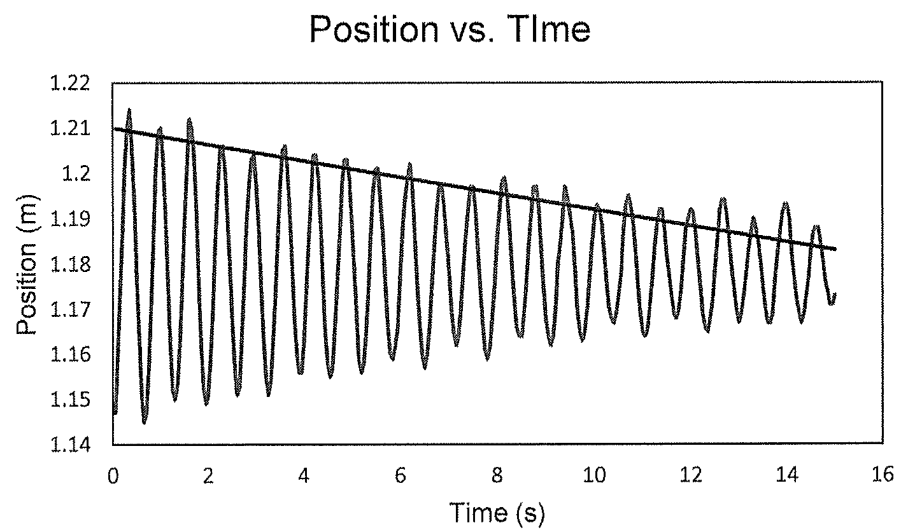

Figure 9: Linear least squares fit plotted over the uncorrected position [redacted] time. Initial amplitude [redacted] and damping coefficien[redacted] ← units?

γ has units as ω.

QUESTION 6: DETERMINE THE ERROR IN γ.

Since we only used the first half ($x = Ae^{-\gamma t}$) of the equation $x = Ae^{-\gamma t}\cos(\omega t + \varphi)$ to estimate γ, we find that $\gamma = \frac{1}{t}\left(\ln\left(\frac{x}{A}\right)\right)$. We took δt to be 0, A to be 1.21 m, and x to be 0.35 m (the position where initial amplitude A occurs). Therefore, we found [redacted] ← units?

$$\delta\gamma = \frac{\left(\gamma * \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta A}{A}\right)^2}\right)}{\frac{x}{A}} =$$ [redacted]

The resultant phase space plot for damped motion is an inward spiral (Figure 10). As motion is damped, velocity decreases and so does amplitude, so they become smaller relative to one another and spiral inward.

Figure 10:

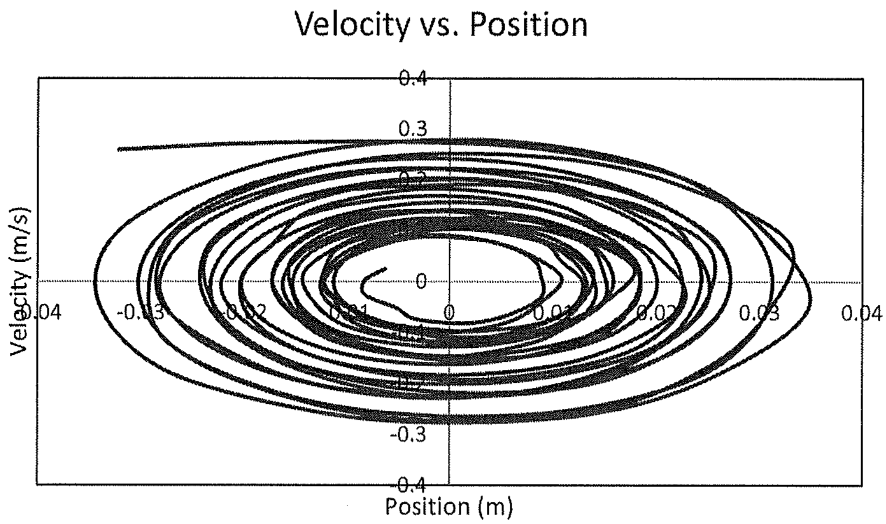

Figure 10: Velocity vs position phase space plot.

Conclusion

By measuring the mass-spring system's undamped motion using LoggerPro, we were able to calculate the spring constant of the system *k* to be [redacted] From here, we were able to measure the period of oscillation of an undamped 500 g mass to within [redacted] of the theoretical period. We found that acceleration and force are out of phase with position in undamped motion. Furthermore, in examining underdamped motion of the same [redacted] mass, we found that the amplitude of oscillation decayed exponentially with a damping constant [redacted]