Magnetic monopole and string excitations in a two-dimensional spin ice

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We study the magnetic excitations of a square lattice spin-ice recently produced in an artificial form, as an array of nanoscale magnets. Our analysis, based upon the dipolar interaction between the nanomagnetic islands, correctly reproduces the ground-state observed experimentally. In addition, we find magnetic monopole-like excitations effectively interacting by means of the usual Coulombic plus a linear confining potential, the latter being related to a string-like excitation joining the monopoles indicating that the fractionalization of magnetic dipoles may not be so easy in two dimensions. These findings contrast this material with the three-dimensional analogue, where such monopoles experience only the Coulombic interaction. We argue, however, that the string looses its tension due to entropic effect and then, free magnetic monopoles may also be found in lower dimensional spin ices.

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Geometrical frustration among spins in magnetic materials can lead to a variety of cooperative phases such as spin glass, spin liquid and spin ice behaving like glass, liquid and ice in nature. The description and understanding of such states are becoming increasingly important not only in condensed matter but also, in other branches like field theories. In a crystal at low temperature excitations above the ground state often behave like elementary particles carrying a quantized amount of energy, momentum, electric charge and spin. Several of these objects arise as a result of the collective behavior of many particles in a material which is most effectively described in terms of the fractions of the original particles. The emergence of these excitations is an example of the phenomenon known as "fractionalization". This occurrence is often tied to topological defects [1] and is common in one-dimensional systems (polyacetylene, nanotubes, etc). In two spatial dimensions the only confirmed case is the involvement of quasi-particles with one-third of an electron's charge in the fractional quantum Hall effect in strong magnetic fields. Among several suggestions, there is also the proposal that the merons forming a skyrmion in two-dimensional (2d) Heisenberg antiferromagnets are spinons and therefore, they are neutral spin-half excitations [2,3]. More recently, the first example of fractionalization in three-dimensional systems was provided in spin ice materials [4]: this is the famous magnetic monopole. Despite some exciting suspicions for its existence from the realms of quantum mechanics, a single magnetic pole remains elusive after decades of searching in particle accelerators and cosmic rays. Very recently, Castelnovo et al.[4] indicated an unexpected but, perhaps, better place to look. Under certain conditions, spin ice magnets behave like a gas of independent magnetic poles. There is even a phase transition at which a thin vapor of these poles condenses into a dense liquid. The existence of these excitations in a condensed matter system is exciting in itself. Our aim in this letter is to study spin ice materials, but in two spatial dimensions. Such structures have



FIG. 1: (Color online) The two-dimensional square lattice studied in this work. Only a few islands are shown. The arrows inside the islands represent the local dipole moments $(\vec{S}_{h(i)} \text{ or } \vec{S}_{v(i)})$.

been artificially produced in a geometrically frustrated lattice of nanoscale ferromagnetic islands [5,6,7]. Here, we examine the excitations ("magnetic monopoles") and how they interact in this 2d system.

Three-dimensional spin-ice materials have the pyrochlore structure in which magnetic rare-earth ions form a lattice of corner-sharing tetrahedra. To minimize the spin-spin interaction energy, the ice rules are manifested: two spins point inward and two spins point outward on each tetrahedron. A similar system was built in two dimensions with elongated permalloy nanoparticles. This artificial material consists of an array of 2d square lattices of the magnetic nano-islands with the largest axis of the islands alternating in orientation along the two principal directions of the array lattice [5]. The magnetocrystalline anisotropy of permalloy is effectively zero, so that the shape anisotropy of each island forces its magnetic moment to align along the largest axis thus, making the islands effectively Ising-like. The intrinsic frustration on this lattice is similar to that in the spin ice model and can be best seen by considering a vertex at which four islands meet. A pair of moments on a vertex can be aligned either to maximize or to minimize the dipole interaction energy of the pair. As shown in Ref.[5], it



FIG. 2: (a) Configuration of the ground-state obtained for L = 6a, in exact agreement with that experimentally observed. Note that the ice rules are manifested at each vertex. This is the case in which the topology demands the minimum energy (see Fig. (3)). (b) Another configuration also respecting the ice rule, but displaying a topology which costs more energy.

is energetically favorable when the moments of a pair of islands are align so that one is pointing into the center of the vertex and the other is pointing out (red islands in Fig. 1) while it is energetically unfavorable when both moments are pointing inward or both are pointing outward (blue islands in Fig. 1). This artificial system exhibits short-range order and ice-like correlations on the lattice, which is precisely analogous to the behavior of the spin ice materials. Here, we consider an arrangement alike that experimentally investigated in Ref. [5]. In our scheme the magnetic moment ("spin") of the island is replaced by a point dipole at its center. To do this, in each site (x_i, y_i) of a square lattice two spin variables are defined: $\vec{S}_{h(i)}$ with components $S_x = \pm 1, S_y = 0$ located at $\vec{r}_h = (x_i + 1/2, y_i)$, and $\vec{S}_{v(i)}$ with components $S_x = 0$, $S_y = \pm 1$ at $\vec{r}_v = (x_i, y_i + 1/2)$. Therefore, in a lattice of volume L^2 one gets $2 \times L^2$ spins (see Fig. 2). Representing the spins of the islands by \vec{S}_i , which can assume either $\vec{S}_{h(i)}$ or $\vec{S}_{v(i)}$, then the 2*d* spin ice is described by the following Hamiltonian

$$H_{SI} = Da^{3} \sum_{i \neq j} \left[\frac{\vec{S}_{i} \cdot \vec{S}_{j}}{r_{ij}^{3}} - \frac{3(\vec{S}_{i} \cdot \vec{r}_{ij})(\vec{S}_{j} \cdot \vec{r}_{ij})}{r_{ij}^{5}} \right], (1)$$

where $D = \mu_0 \mu^2 / 4\pi a^3$ is the coupling constant of the dipolar interaction and *a* is the lattice spacing. The sum is either over all $2 \times L^2 - 1$ pairs of spins in the lattice for the case with open boundary conditions (OBC) or over all spins and their images for the case with periodic boundary conditions (PBC). We study these two possibilities; the first one is more related to the artificial spin ice fabricated in Ref.[5], while using PBC we minimize the border effects. In the system with PBC the Ewald summation [8,9] is used.

We consider first the ground states obtained from Hamiltonian (1) describing the 2d spin ice. To do this we use a process known as simulated annealing [10], which is a Monte Carlo calculation where the temperature is



FIG. 3: The 4 distinct topologies and the 16 possible magnetic moment configurations on a vertex of 4 islands. Although configurations (a) and (b) obey the ice rule, the topology of (a) is more energetically favorable than that of (b). Hamiltonian (1) correctly yields to the true ground-state based on topology (a), without further assumptions. Topologies (c) and (d) does not obey the ice rule. Particularly, (c) implies in a monopole with charge Q_M .



FIG. 4: (Color online) The two basic shortest strings used in the separation process of the magnetic charges: pictures (1) and (2) exhibit strings 1 and 2 respectively. The left circle (red) is the positive charge (north pole) while the right circle (black) is the negative (south pole).

slightly reduced in each step of the process in order to drive the system to the global minimum. Several tests for systems with different sizes L ($6a \leq L \leq 80a$) were studied. The final configuration (ground state) was found to be twofold degenerate (see part (a) of Fig. 2 fora lattice with L = 6a). If we consider the vorticity in each plaquette, assigning a variable $\sigma = +1$ and -1 to clockwise and anticlockwise vorticities respectively, the ground state looks like a checkerboard, with an antiferromagnetic arrangement of the σ variable. Note that the ground state clearly obeys the ice rule. We remark that it is impossible to minimize all dipole-dipole interactions. On each vertex there are six pairs of dipoles and only four of them can simultaneously minimize the energy. It is important to mention that, although there are other possible configurations that also obey the ice rule, these are not the ground state. Indeed, the state shown on the right side of Fig. 2 has energy about four times larger than that of the ground state. The difference between these two states is related to the distinct topologies for the configurations of the four moments (see Fig. 3). It was experimentally shown in Ref. [5] that, while the topologies of types (a) and (b) obey the ice rule, the case (a) has smaller energy than case (b). Our theoretical calculations confirm this fact. The same ground state was also reported in Refs. [6,7]

The system is, therefore, naturally frustrated. In



FIG. 5: (Color online) Inset: the interaction potential between two magnetic charges (with opposite signs) as a function of r = R/a. The baseline of V(r) is also plotted: the curves are obtained by fitting the data to $\alpha R + \beta$ and q/R + b'X + c minus $\alpha R + \beta$.

the two-in/two-out configuration, the effective magnetic charge $Q_M^{i,j}$ on each vertex (i,j) is zero. The most elementary excited state involves inverting a single dipole to generate localized "dipole magnetic charges", which implies in a "vortex-pair annihilation". Such inversion corresponds to two adjacent sites with net magnetic charge $Q_M^{i,j}$, which is alike a nearest-neighbor monopoleantimonopole pair. In principle, such "monopoles" can be separated from one another without violations of local neutrality by flipping a chain of adjacent dipoles. Therefore, it should be interesting to study how these excitations interact in two dimensions. Using OBC for a while and considering the presence of a single dipole near zero temperature, we then start to separate the magnetic charges: such a process inevitably leads to a stringlike excitation joining these poles, whose presence is evidenced by an extra energy cost behaving as bX, where X is the length of the string and b > 0 is the effective string tension. Our calculations then indicate that the total energy cost of a monopole-antimonopole pair separated by R is thus the sum of the usual Coulombic term, roughly equal to q/R (q is a constant), and a term roughly equal to bX resulting from the string binding the monopoles (there is, of course, also an *R*-independent value associated to the creation of the pair). We remark that the energy cost of the string has some dependence on the way that the monopole-antimonopole is separated. In order to establish a link between the monopole-antimonopole distance R and the string length X we choose two basic string shapes to move the charges as shown in Fig. 4. Of course, the shortest strings will be formed around the straight line joining the monopoles and, therefore, we choose two different ways in which they may be created as the charges are separated (see Fig. 4). Firstly, using the string shape 1 and starting in the ground state we choose an arbitrary site and then the grev spins in Fig. 4 are flipped, so creating a monopole-antimonopole separated by R = 2a. In sequence, the spins marked in blue are flipped and the separation distance becomes R = 4a

and so on. In this case X = 4R/2. Note that the string surges in the system because in the separation process, the topology is locally modified, although still keeping the ice rule; in the region between the two poles, the topology of type (b), which has larger energy than that of type (a), prevails. Being essentially localized along the line joining the monopoles this additional amount of energy increases as the distance between the magnetic charges increases, justifying the bX term. Until now we have only considered the shortest strings connecting two poles. However, many dipole strings of arbitrary shape and size can be identified that connect a given pair of monopoles. The associated energy cost increases with Xand diverges with the length of the string. So, at first sight, the monopoles should be confined in the artificial material. As we will argue later, it is possible that the string tension vanishes at a critical temperature proportional to b and hence, free magnetic monopoles may also be found in the 2d system. Now, it is important to mention that the combined objects monopole + string can be constructed without violating the ice rule. It is similar to what occur in the Standard Model of electroweak interactions: combined objects monopole + string (sometimes called "nexus" [11]) can be constructed without violation of the condition $\vec{\nabla} \cdot \vec{B} = 0$. In this case, the magnetic monopole looks like a Dirac monopole, but the Dirac string is physical and is represented by the cosmic string.

The potential V(r) (the energy of the excited configuration minus the energy of the ground state) as a function of r = R/a is shown in the inset of Fig. 5 for the string shape presented in part (1) of Fig. 4. The behavior is apparently linear but the function $f_a(R) = q/R + b'R + c$, with $q \approx -0.00122 Da, b = b'/2 \approx 0.00305 D/a, c \approx$ 0.00734D, fits better the data than the purely linear possibility $q(R) = \alpha R + \beta$ with $\alpha \approx 0.00611 D/a, \beta \approx$ 0.00702D. This difference becomes clearer when we analyze the χ^2/Dof which is equal to 1.04×10^{-8} for the linear fitting and 4.5×10^{-13} for the fit with $f_q(R)$. Also in Fig. 5 we draw a baseline of the potential using the linear fit. One can clearly see that the $f_q(R)$ describes better the data and, therefore, $V(r) \approx f_q(R)$. The same method was repeated using the string shape 2. In this case, the charges are separated diagonally and $X = 2R/\sqrt{2}$. The results are qualitatively the same and the values of the constants are: $q \approx -0.00125 Da, b = b'/\sqrt{2} \approx$ $0.00317D/a, c \approx 0.00724D$. Note that the quantitative changes are small. The results are also qualitatively the same if PBC are used instead OBC. Furthermore, quantitative differences between PBC and OBC calculations are smaller than 1% for constants b and c, while it is smaller than 9% for q. The larger difference for constant q can be understood if one remembers that the use of PBC will imply that the charges interact also with their images.

To have an idea of the value of the magnetic charge Q_M , we consider the usual expression for the Coulombic interaction (in MKS units) $-\mu_0 Q_M^2/4\pi R$ to com-

pare with q/R. Thus, $\mid q \mid = \mu_0 Q_M^2/4\pi$, or $Q_M \approx$ $\pm \sqrt{4\pi |q|/\mu_0} \sim 0.035 \mu/a$. Using the data of Ref. [5] (such as $a \sim 320 nm$ and $\mu \sim 2.79 \times 10^{-16} JT^{-1}$) to estimate the fundamental magnetic charge of an excitation in the array of ferromagnetic nano-islands, we get $Q_M \approx 3 \times 10^{-11} Cm/s$, which is about 6×10^3 times smaller than the fundamental charge of the Dirac monopole $(Q_D = 2\pi\hbar/\mu_0 e)$. Such charge can even be tuned continuously by changing the lattice spacing. We notice that the value obtained for the magnetic charge in two dimensions is proportional to $\pm \mu/a$ but the proportionality constant is not 1 as in the usual threedimensional definition. Indeed, this difference is related to the 2d character of the system; for instance, the charge distribution of a pole is essentially planar with a superficial density $\sim Q_M/a^2$.

Before concluding, it is important to analyze the behavior of the string tension as some parameters are varied in the system. The string tension for the artificial system built in Ref^5 is approximately given by $b \approx 2.26 \times 10^{-15} J/m \approx 4.5 \times 10^{-3} eV/a$. Therefore, it is necessary a relatively large amount of energy (about $10^{-3}eV$) to separate the "two-dimensional monopoles" by one lattice spacing, regardless of how far apart they are. Consequently, at low temperature, there is insufficient thermal energy to create long strings, and so the "monopoles" would be bound together tightly in pairs. The string tension can be artificially reduced by increasing the parameter a $(b \propto 1/a)$. However, it has also the effect of decreasing the magnetic charge since Q_M is proportional to 1/a. A manner of reducing b without affecting Q_M is increasing the temperature. By using the random walk argument, one can see that the many possi-

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ble ways of connecting a pair of monopoles with a string give rise to a string configurational entropy proportional to R. Then, crudely speaking, as the temperature increases, the string tension should decrease like $b - \epsilon k_B T$, with $\epsilon = O(a)$. It means that the string looses its tension by entropic effect and, therefore, it should vanish at some critical temperature T_c on the order of b. Thus, above T_c the deconfinement occurs and magnetic monopoles may exist freely in 2d spin ices. This phenomenon also happens with half-vortices in physical systems described by 2d classical XY models [12] such as superconductors. Once the deconfinement realizes, the question of technological applications of this system is relevant. For instance, learning how to move the magnetic monopoles around would be an important step towards technologies involving magnetic analogous of electric circuits.

Finally, we would like to say that the above scenario involving the monopole interactions may be drastically changed if one considers these excitations on the configuration (b) of Fig. 2. As experimentally shown in Ref.[6], this metastable state is a very real possibility when magnetic fields are applied. In this case only the topology of type (b) of Fig. 3 is present in the separation process. Nevertheless, the string tension should be zero and monopoles could exist freely even at low temperatures. Another point that deserves to be remarked about these 2d spin ice systems is that they may display a very rich thermodynamical behavior since the presence of different topological defects such as monopoles, strings and vortices can affect the phase transitions in an unexpected way.

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