THEORY OF RANDOM MAGNETS

After almost a decade of intense research on their unusual phases and even more unusual dynamical behavior, random magnets have emerged as prototypes for a wide variety of systems with frozen-in disorder.

Daniel S. Fisher, Geoffrey M. Grinstein and Anil Khurana

Much of the enormous increase in our understanding of collective phenomena during the past few decades has arisen from the study of magnetic systems. The formulation of concepts such as universality, broken symmetry, and scaling near continuous phase transitions, as well as the development of the powerful ideas of the renormalization group, have been strongly influenced by research in magnetism. This is due in part to the availability of a host of experimentally accessible magnetic systems, and in part to the remarkable fact that simple models of magnetism capture the essential physics of the phases and ordering transitions in more complicated systems.

Until relatively recently, study of the collective behavior of magnets and other condensed matter systems has centered largely on ideal pure materials, such as perfect crystals. Quenched, or frozen-in, structural disorder is so ubiquitous, however, that it affects the properties of virtually all experimental systems to some degree. Experimentalists are often driven to elaborate lengths to reduce the randomness in their samples to negligible levels. Only in the past ten years or so has widespread appreciation for the fascinating phenomena caused by disorder itself developed in the condensed matter community. No longer denigrated as "dirt," "junk" or an unavoidable nuisance, randomness and its consequences have become the objects of intense study.

Disordered magnets have emerged as prototypes for collective phenomena in systems with quenched disorder. Again, this is a result of the simplicity of magnetic models and the existence of many convenient experimental realizations, some of which have almost ideal, homogeneous randomness. Ideas from the study of random magnets have already been applied to structural phase transitions and charge-density waves in random alloys, to the melting of intercalates, to dirty superconductors, to fluids and superfluids in porous media and to adsorption and wetting on disordered surfaces. The renormalization group, conceived by K. G. Wilson to handle problems involving a broad range of length and time scales,[1] provides, as it does for pure systems, the unifying framework for the understanding of this marvelously diverse class of random systems.

In this article we summarize current understanding of the phases and phase transitions of random magnets, comparing and contrasting them with conventional, pure magnets. We first briefly review the equilibrium phases and phase transitions of pure magnets, emphasizing the role of symmetries and the spatial dimensionality. Identifying three main classes of disordered magnets—random exchange, random field and spin glass—we then consider the central issues connected with their equilibrium properties, such as the existence and nature of their ordered phases. We will see that for spin glasses, a

Daniel S. Fisher was until recently a member of the technical staff at AT&T Bell Labs, Murray Hill, New Jersey. He is now a professor of physics at Princeton University. **Geoffrey M. Grinstein** is a physicist at IBM Thomas J. Watson Research Center, Yorktown Heights, N. Y. **Anil Khurana** is an associate editor of PHYSICS TODAY.

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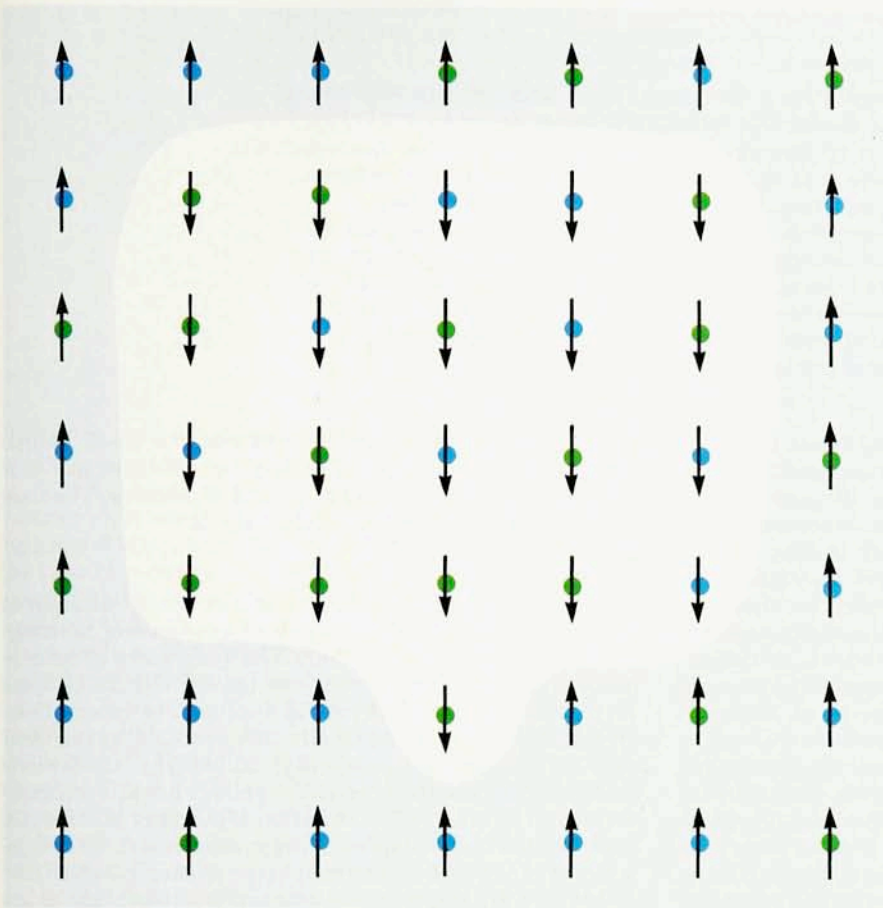


Droplet of overturned spins (white) in a putative ferromagnetic (up) ground state of the random-field Ising magnet. A frozen-in magnetic field acts at each site in the magnet; the field points up on the blue sites and down on the green sites. The configuration with the overturned droplet can have lower energy than that of the wholly ferromagnetic up configuration because the droplet occurs in a region in which the random fields point predominantly down. Y. Imry and S.-k. Ma estimated the probability of occurrence of such droplets and concluded that the ferromagnetic state is unstable with respect to the formation of large droplets in fewer than 2 dimensions even at $T = 0$.

surprising degree of controversy still surrounds these questions. Finally we turn to nonequilibrium effects, which distinguish random magnets strikingly from their pure counterparts. The existence of a rather sharp "freezing" temperature, below which the system responds extremely slowly to changes in external conditions, is a dramatic signature of quenched randomness on the ordering process. Below this temperature, many random systems are simply unable to achieve equilibrium in any reasonable time and so show history-dependent behavior. We will argue that in random magnets, unlike in pure systems, such nonequilibrium manifestations are often inextricably linked to equilibrium collective phenomena. This necessitates a refinement of such common notions as equilibrium states and metastability.

Pure magnets

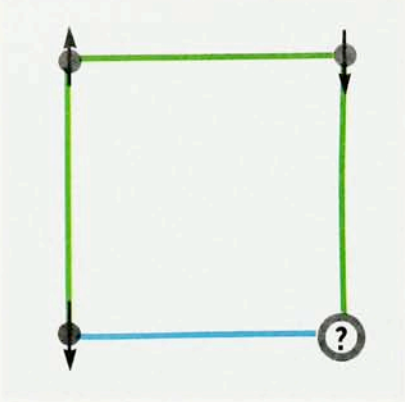
We focus on the simplest magnetic models, those consisting of Ising spins, S_i, which can only point "up" ($S_i = +1$) or "down" ($S_i = -1$). The basic Ising Hamiltonian consists of exchange interactions J_{ij} between nearest-neighbor pairs of spins (i, j) on a hypercubic lattice in d spatial dimensions:

$$\mathsf{H} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j \qquad (1)$$

This Hamiltonian exhibits an obvious, global spin-flip symmetry: The energy is invariant under a simultaneous change of sign of all the spins.

In a pure Ising ferromagnet all the exchange interactions have the same value J. Such a system has two phases: At temperatures large compared with J, the entropy dominates over the energy and the spins fluctuate almost independently.[1] The global spin-flip symmetry is preserved in this, the paramagnetic phase, each spin being up on average as often as it is down. Correlations between two spins separated by a distance r decay like $e^{-r/\xi}$, where ξ is a characteristic correlation length. Conversely, at low temperatures, the energy dominates over the entropy and the spins are almost all aligned, either in the up or down direction. In this, the ferromagnetic phase, then, there are two equivalent equilibrium states—"up" and "down"—related to each other by the spin-flip symmetry. In choosing one of these states over the other the system spontaneously breaks this symmetry, and each spin develops a nonzero expectation value $\langle S_i \rangle$, which indicates that in equilibrium it spends more time pointing up, for example, than down. (Here $\langle \ldots \rangle$ denotes an average over very long times.) This "spontaneous magnetization density," $m \equiv \langle S_i \rangle$, vanishes in the symmetric, high-temperature phase and therefore serves as an order parameter that characterizes the broken-symmetry, or ordered, phase. In the ordered phase the correlation between two spins approaches the non-zero value m^2 as the distance between them goes to infinity. This property is called long-range order. The transition between the two phases occurs at a critical temperature T_c of order J, at which m vanishes when the system is heated from low temperatures. In the pure Ising model of equation 1, ξ diverges and m vanishes algebraically at T_c, a behavior characteristic of so-called continuous phase transitions.

This simple intuitive picture of the phases of the pure Ising ferromagnet is correct for all dimensions $d > 1$. In one dimension, the fact that each spin is connected to the



Frustration in spin glasses arises because loops with an odd number of antiferromagnetic bonds do not have a unique spin configuration of lowest energy. In a simple square with three antiferromagnetic bonds (green) and a ferromagnetic bond (blue), all of equal strength, either of the two orientations of the spin at the lower right corner gives the same energy when the orientations of the other three spins are fixed as shown. This energy is higher than that of the lowest-energy configuration of a square loop with *even* numbers of antiferromagnetic and ferromagnetic bonds of equal strengths.

rest of the lattice by only two bonds means that thermal fluctuations can readily disrupt any ferromagnetic alignment. Thus, while at zero-temperature the one-dimensional Ising model is ordered, at any positive temperature the entropy always dominates the energy at long length scales and the system is disordered. The principle that ordered states are more easily destroyed by thermal fluctuations as d decreases is very general and gives rise to the notion of the lower critical dimension, d_l, which is defined as the lowest dimension (not necessarily integer!) above which an ordered phase can exist at nonzero temperature. For Ising ferromagnets, then, $d_l = 1$.

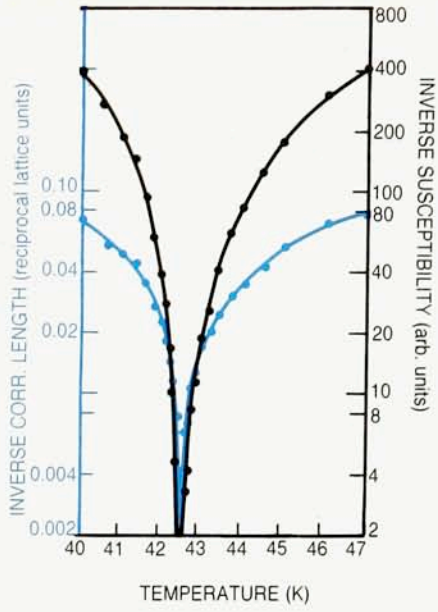
One can often usefully describe ordered phases in terms of their ground states and excitations. In an Ising ferromagnet the two obvious ground states have all spins pointing either up or down. (See the box on page 62, however.) The lowest-energy excitations of linear size L are connected, compact droplets of L^d overturned spins. It costs energy, of order J times the surface area $\sim L^{d-1}$, to create such a droplet. This simple idea underlies the important notion of the "stiffness" of an ordered phase—in many ways its most fundamental characteristic, and a particularly vital one in our understanding of disordered systems. One gets a sense of how basic this notion is by observing that nonphysicists who have never heard of broken symmetry have a clear intuitive understanding that solids are distinguished from liquids by their rigidity, or resistance to distortion. J. M. Kosterlitz and D. J. Thouless have shown that certain ordered phases—for example, solids in two dimensions—can exist without broken symmetry or long-range order![2] Such phases are characterized by their stiffness; for example, a two-dimensional solid is characterized by a nonzero shear modulus.

The stiffness of a system depends on the length scale on which it is probed; a rough definition of the stiffness at length scale L is the cost in free energy of distorting the order significantly in a region of volume L^d. In an Ising ferromagnet, a domain wall separating up and down spins is needed to distort the order. This costs a free energy proportional to the wall's area L^{d-1}, the coefficient of proportionality being the interfacial tension σ. At $T = 0$, $\sigma = J$. In general, the free-energy cost of excitations and hence the stiffness of an ordered phase grows with length scale L like L^θ, where θ is the stiffness exponent. If θ is negative, large excitations have free energies much less than $k_B T$ and so proliferate, destroying the ordered phase. For an ordered phase to exist, therefore, θ, which depends on the dimension d as well as on the system under consideration, must be non-negative. The marginal case $\theta = 0$ thus corresponds to the lower critical dimension. In one dimension, for example, ferromagnetic order in pure Ising models is destroyed by droplet fluctuations because $\theta = d - 1$. For solids and superfluids $\theta = d - 2$, which implies that these ordered phases cannot exist in fewer than two dimensions. The special Kosterlitz–Thouless phases occur in the marginal case $d = 2$.

Disorder

Most random magnets are substitutionally disordered materials in which several kinds of magnetic or nonmagnetic ions are alloyed together. The qualitative effect is to make the exchange interactions between pairs of spins vary randomly from one pair to another, the interactions for different pairs being totally uncorrelated in the ideal case. The simplest possibility, called the random-exchange model, occurs when the couplings are all ferromagnetic (that is, all $J_{ij} > 0$ in equation 1) but vary in strength. Spins are still fully aligned in the ground states of this system; for $d > d_l = 1$ a ferromagnetic phase, in which the global spin-flip symmetry is spontaneously broken, exists below a finite critical temperature. Though early experimental data suggested that the sharp transitions in pure materials were somehow broadened, or "smeared," by spatial inhomogeneities, heuristic arguments by A. B. Harris, later confirmed by renormalization group calculations, showed[3] that this need not, in fact, be the case, at least in the absence of macroscopic inhomogeneities, or long-range correlations in the impurity positions. To see this, imagine dividing the system into blocks of ξ^d spins, where ξ is the correlation length. Spins in different blocks can be thought of as essentially uncorrelated. As ξ grows, the statistical variations in the average exchange strength (and hence in the effective T_c) of the various blocks shrinks, thus allowing the system to undergo a perfectly sharp transition at a well-defined T_c. Fabrication of high-quality crystals with almost ideal substitutional disorder has since allowed experimental confirmation of this idea.[4] (Indeed, the improvement in fabrication techniques has sparked significant overall progress in the understanding of random systems.) The earlier, "smeared" transitions were presumably artifacts of macroscopic inhomogeneities. In fact, the phase transitions in the best random magnets are virtually as sharp as those in pure magnets (see the figure on the opposite page). Thus the equilibrium behavior of random-exchange ferromagnets is qualitatively the same as that of pure systems.

Far more interesting behavior occurs when the distribution of the J_{ij}'s includes both positive (ferromagnetic) and negative (antiferromagnetic) interactions. It is then possible for the interactions to compete with each other as shown in the figure above, making it impossible to satisfy simultaneously all of the exchange interactions of the Hamiltonian. This property, called frustration, is responsible for much of the fascinating behavior of random systems. Finding the ground states of a frustrated system



Sharp divergences in the correlation length and the staggered susceptibility at a critical T_c mark the transition to an antiferromagnetic state in the disordered antiferromagnet $Fe_{0.5}Zn_{0.5}F_2$. The figure shows the inverse of the correlation length (blue) and of the staggered susceptibility (black). (Adapted from reference 4.)

is a highly nontrivial optimization problem in which one must decide which bonds to satisfy.

If the density of negative couplings, and hence the frustration, is small, only a few spins will be misaligned in the ground state, and the ordered phase will generally be like that of a normal ferromagnet. When enough of the couplings are negative to produce strong competition, however, the behavior becomes very subtle, and basic issues such as the existence and nature of an ordered phase become difficult. Such systems are known as spin glasses.[5] The system defined by the Hamiltonian of equation 1 but with the J_{ij}'s symmetrically distributed about zero is the Ising-spin version of the model first proposed by S. F. Edwards and P. W. Anderson as a prototypical spin glass.[6] (Their original model involved Heisenberg spins—unit-length vectors free to point in any direction on the unit sphere—which are more realistic.)

Rather than immediately considering spin glasses, we first discuss the random-field Ising model, a somewhat simpler system with a different kind of frustration. After years of sometimes bitter controversy, a rather clear understanding of this model has finally emerged. (Space limitations prevent us from discussing random-axis models, an interesting class of frustrated systems in which each of the Heisenberg spins experiences single-site anistropy, the orientation of the preferred axes varying randomly from site to site. These magnets have features in common with both random-field magnets and spin glasses. We refer the interested reader to reference 5 for a brief review and bibliography.)

Random-field Ising model

The model is defined by the Hamiltonian

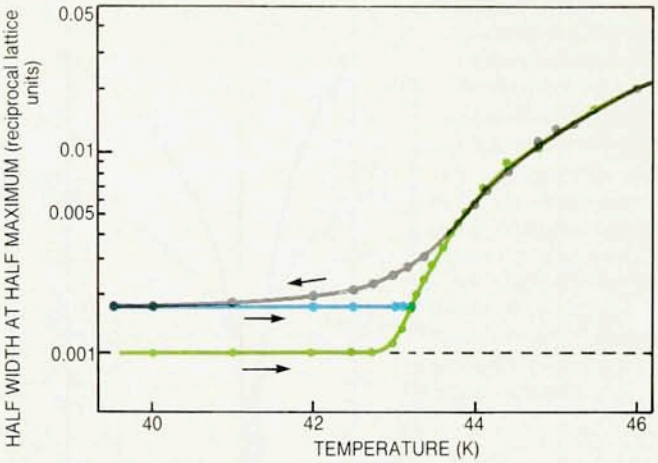
$$H_{RF} = -J\sum_{\langle i,j\rangle} S_i S_j - \sum_i h_i S_i \qquad (2)$$

The magnetic fields h_i are random variables with no (or at most, short ranged) correlations between their values at different sites. They are chosen from an even distribution, so the fields point randomly up or down and their average value is zero. Though the random-field model remained a theoretical construct for several years following its introduction[7] in 1975, it is now understood to describe the essential physics of a strikingly rich class of experimentally accessible disordered systems. These include structural phase transitions in random alloys, commensurate charge-density-wave systems with impurity pinning, binary fluid mixtures in random porous media, and the melting of intercalates in layered compounds such as TiS_2. The most accurate data thus far have come from a magnetic realization—disordered Ising antiferromagnets such as $Fe_xZn_{1-x}F_2$ in a uniform magnetic field.

The behavior of the random-field model is governed by the competing tendencies of the spins to align ferromagnetically under the impetus of the exchange J or to follow the local fields h_i and so be uncorrelated. This is a competition between two energies—the exchange energy and the random-field energy—and therefore differs from the energy–entropy battle that controls the phase transitions in pure and unfrustrated random systems. Thermal fluctuations play a secondary role in this struggle, since, as we shall see, their disordering effect is considerably weaker than that of the random fields. One therefore can understand most of the equilibrium physics of the model at all temperatures, including the critical properties, by studying the model at zero temperature.

The random-field model's simplicity relative to spin glasses stems from the fact that its possible phases and order parameters seem rather straightforward. Even though the Hamiltonian of equation 2 has no spin-flip symmetry, the model is spin-flip symmetric in a statistical sense: The transformation $h_i \to -h_i$, $S_i \to -S_i$ shows that the magnetization m, averaged over the ensemble of random fields, vanishes for any even distribution of the h_i's. Thus $\langle m\rangle_{av}$ is the natural order parameter for the random-field model: When the exchange dominates, the system orders ferromagnetically, and $\langle m\rangle_{av}$ acquires a nonzero value. When random-field or thermal effects dominate, the system is a paramagnet, with $\langle m\rangle_{av} = 0$, as required by the "average" spin-flip symmetry.

This apparent simplicity is deceptive, however. Let us denote the typical magnitude of the random fields by h. One naively expects that at low temperatures the system will be a paramagnet for $h/J \gg 1$ and a ferromagnet for $h/J \ll 1$. While it is true that for sufficiently large h/J each spin will follow its local field, producing paramagnetism at all temperatures and in any dimension, the situation at small h/J is more subtle. This was first appreciated by Y. Imry and S.-k. Ma,[7] who studied the stability, with respect to the overturning of a large droplet of spins with linear size L, of an assumed perfectly aligned ferromagnetic ground state (say "up") in the presence of random fields $h \ll J$. Formation of such a droplet will gain energy from the random-field term of the Hamiltonian only if statistical variations have produced a preponderance of down-pointing fields within it (see the figure on page 57). The excess of down fields will typically be of order $(L^d)^{1/2}$ in d dimensions, whereupon roughly $hL^{d/2}$ in random-field energy is gained. The exchange-energy cost of the droplet



Hysteretic behavior of a random-field Ising magnet. The figure shows the width at half maximum, proportional to the inverse correlation length, of a neutron scattering peak in the disordered antiferromagnet $Fe_{0.6}Zn_{0.4}F_2$ placed in a 5.5-T magnetic field. When the magnet is cooled *in* the field, the half width (gray) is larger than the experimental resolution (dashed line); but if the field is turned on *after* the magnet is cooled below $T_c \approx 43$ K, then the half width is limited by experimental resolution (green). A resolution limited half width is evidence for antiferromagnetic ordering. For $T < T_c$ the half width remains essentially constant at its low-temperature value (blue) when the magnet is heated after being cooled in the field . [Adapted from D. P. Belanger, S. M. Rezende, A. R. King and V. Jaccarino, *J. Appl. Phys.* **57**(1) 3294 (1985).]

is proportional to its surface area, $\sim JL^{d-1}$; thus for any $d > 2$ it dominates the gain from the random-field term at large L, ensuring the stability of the ferromagnetic ground state. When $d < 2$, however, the random-field energy dominates at sufficiently large L even for arbitrarily small h/J, thereby rendering ferromagnetism unstable with respect to droplet formation. This domain-wall argument therefore predicts that the lower critical dimension is 2 for the random-field model when h/J is small. For $d = 2$, where both energies are proportional to L, the exchange energy dominates for $h/J \ll 1$, but a more sophisticated version of the argument that treats the meandering (or roughening) of walls in response to local pockets of fields pointing predominantly up or down (see the figure on page 57) shows that the ferromagnetic ground state is actually unstable in this marginal case.[8]

Doubt was soon cast upon the conclusion $d_l = 2$ by formal field-theoretic arguments called dimensional reduction,[8] which purported to show that $d_l = 3$. (See the box on page 67.) Fueling the ensuing controversy were results of quantitative experimental studies of random-field magnets made possible by the fabrication of extremely high-quality dilute Ising antiferromagnets such as $Fe_xZn_{1-x}F_2$. Application of a uniform magnetic field to such systems generates an effective random field that couples to the antiferromagnetic order parameter, producing a realization of the random-field model in which the strength of the effective random field varies with the externally applied uniform field.[9] In principle, high-resolution neutron scattering experiments on these systems ought to have settled the debate about d_l, in that the development of long-range antiferromagnetic order (that is, a Bragg peak in the static structure factor) at low temperatures in three-dimensional samples would confirm that $d_l = 2$. In practice, this expectation was confounded by the occurrence of hysteresis: Samples cooled in even very modest effective random fields showed no Bragg peak and hence no long-range magnetic order, while the long-range antiferromagnetic order developed under cooling in zero applied (that is, zero random) field persisted under subsequent application of even relatively large fields (see the figures above and on the opposite page).[10] The same basic phenomenology was observed in two- and three-dimensional samples. Difficulties in deciding which, if either, of the two modes of measurement was yielding the true equilibrium behavior extinguished hopes for a clean experimental resolution of the d_l quandary; indeed, these problems intensified the debate.

More recently, rigorous proofs by J. Z. Imbrie and by J. Bricmont and A. Kupiainen have established the correctness of the result $d_l = 2$, and hence of the equilibrium phase diagram of the three-dimensional random-field model shown on the opposite page.[11] Furthermore, an explicit mechanism for the failure of dimensional reduction, connected with the neglect of statistically rare but physically significant regions of the sample (see the box on page 67) has now been identified.[12] This still leaves unexplained, however, the apparent inability of the experimental systems to equilibrate on reasonable time scales. We shall defer treatment of this fundamental question to the section on nonequilibrium phenomena.

Spin glasses

Theoretical interest in spin glasses was initiated by the experimental observation[5,13] of a rather sharp cusp in the temperature dependence of the low-field, low-frequency susceptibility in the dilute metallic alloy CuMn (see the figure on page 63), at roughly 1% concentration of magnetic ions (Mn). (See the columns by Anderson in the January, March, June and September issues of PHYSICS TODAY.) Neutron diffraction studies showed that no Bragg peak at any wavenumber arises below the temperature T_f where this cusp occurs; thus no long-range ferromagnetic or antiferromagnetic order accompanies it. As one approaches T_f from above, relaxation times become extremely long and the system begins to exhibit hysteresis, which suggests that T_f is a kind of "freezing" temperature. The figure on page 63 schematically shows a typical history-dependent effect: The susceptibility of samples cooled in a finite field is rather flat below T_f and is almost reversible under subsequent heating and cooling. However, application of the same field to a sample cooled in zero field results in a much-reduced susceptibility at temperatures below T_f; this reduced susceptibility increases extremely slowly with time for fixed field and temperature. Magnetic remanence—the persistence of a finite magnetic moment that decays slowly after the system is field-cooled below T_f and the field is removed—is also a standard feature of spin glasses. Although these phenomena were first observed in dilute metallic samples, the same qualitative features have since been seen in many very different systems, such as the magnetic insulator $Eu_xSr_{1-x}S$ at high concentrations; these characteristics are now taken to constitute a loose definition of spin glass behavior.[13]

Edwards and Anderson attributed these phenomena to the competition between random ferromagnetic and antiferromagnetic interactions.[6] They hypothesized that the cusp and apparent freezing at T_f were associated with a true phase transition into a state with broken spin-flip symmetry. They took the viewpoint that the frustration induced by the competing interactions is paramount and

that the details of how it arises in real systems are peripheral. Thus, even though in dilute metallic alloys like CuMn competing interactions arise because of the oscillations in sign of the long-range (RKKY) interaction between the spins of the randomly positioned magnetic ions, Edwards and Anderson chose a simple lattice model with random, competing short-range interactions—the Heisenberg-spin version of equation 1 with the J_{ij}'s distributed symmetrically about zero. They proposed that the model orders by spontaneously breaking the spin-flip symmetry to produce the low-temperature (spin glass) phase. Each spin S_i develops a nonzero expectation value, $m_i = \langle S_i \rangle$, in this phase, but the sign and magnitude of m_i's vary from site to site because of the random competing interactions. The spatial average of the m_i's, that is, the average magnetization density, thus vanishes, as do all other Fourier amplitudes of the m_i's, consistent with the absence of Bragg peaks in the neutron scattering measurements.

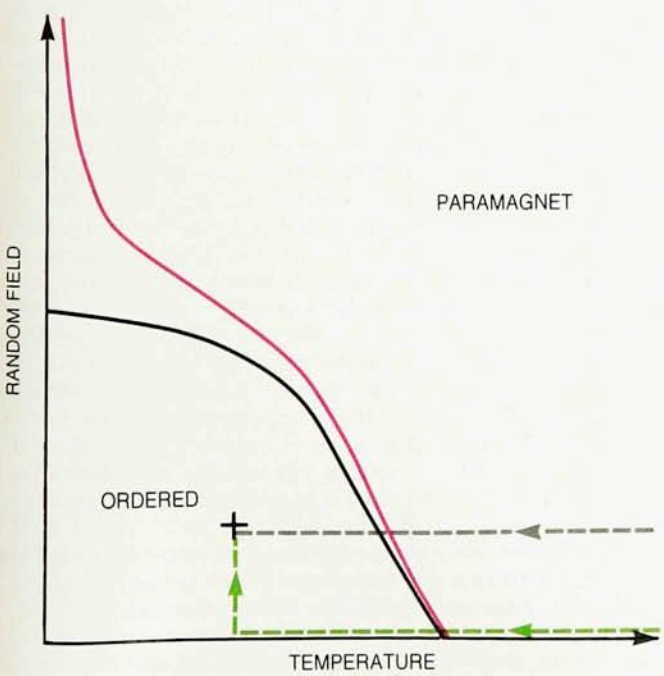
Edwards and Anderson proposed the quantity

$$q \equiv (1/N) \sum_{i=1}^{N} m_i^2$$

where N is the number of spins, as a suitable order parameter. This "Edwards–Anderson order parameter" vanishes in the disordered phase, where $m_i = 0$ by spin-flip symmetry, and is nonzero in the spin glass phase. Edwards and Anderson went on to construct an approximate mean-field solution for their model, finding that q did indeed develop a nonzero value at a continuous phase transition at finite temperature, with an associated cusp in the susceptibility, consistent with the measurements.

This insight left open the crucial question of the existence of the ordered, spin glass phase in the physical dimensionalities 2 and 3, that is, of the lower critical dimension for spin glasses. There are undoubtedly large clusters of spins in the spin glass ground state that, owing to a delicate balance between ferromagnetic and antiferromagnetic interactions, can be flipped with a relatively small energy cost. This notion that there are many configurations differing substantially from the ground state in structure but almost identical to it in energy suggests that the spin glass state is particularly fragile and might be readily destroyed by thermal fluctuations. Unfortunately, computing d_l is extremely hard. The spin glass ground state, in which spins point up or down in a complicated pattern to minimize the exchange energy of the random, competing interactions, is hopelessly difficult to compute analytically, and can be computed numerically only for very small samples. As in the random-field model, one expects droplet excitations of size L away from the ground state to be the chief destabilizing agents of the ordered, spin glass state. Unlike in the random-field model, however, the free-energy cost of such droplets (that is, the stiffness) cannot readily be determined. In consequence, estimates of d_l for Ising spin glasses have wandered inconclusively between 2 and 4 for ten years. In the last few years, however, a fairly universal consensus that $2 < d_l < 3$ has emerged.[5] Much of this progress has come from exhaustive Monte Carlo simulations on samples with up to $64 \times 64 \times 64$ lattice sites in three dimensions. A major advance in the effectiveness of the Monte Carlo method for spin glasses resulted from the decision of A. P. Young and others to study the behavior of the spin glass susceptibility χ_{SG} in the paramagnetic phase as the putative critical temperature T_c is approached from above. Working in the paramagnetic phase circumvents the problem of the very long equilibration times encountered below T_c (see the section on nonequilibrium phenomena). According to the theory of phase transitions, χ_{SG}, an analogue for spin glasses of the staggered susceptibility whose singular behavior characterizes the phase transition in antiferromagnets, should diverge at T_c if the spin glass transition is continuous. Numerical simulations of three-dimensional Ising spin glasses show clear signs of this divergence, so one may infer with some confidence that a continuous phase transition occurs (see the upper figure on page 64). Similar studies in two dimensions, on the other hand, indicate that T_c is pushed down to 0. From this we conclude that $2 < d_l < 3$. Of course this inference, based as it is on the analysis of finite systems, will always be somewhat uncertain. But most workers in the field find the simulations and other numerical evidence[5] in favor of this result quite compelling.

Experimenters have used a similar strategy to determine whether real three-dimensional spin glasses undergo true phase transitions.[5] The nonlinear susceptibility of a spin glass (defined as $\chi_3 \equiv \partial^3 M / \partial H^3 |_{H=0}$, where M is the

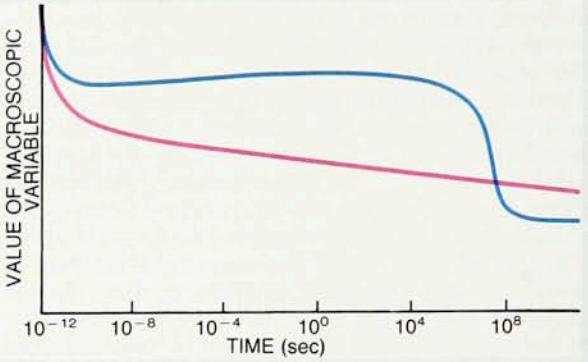


Theoretical phase diagram of three-dimensional random-field magnets. A true equilibrium phase transition occurs at the black line, whereas the red line marks the onset, on cooling, of irreversible (hysteretic) behavior on macroscopic timescales. A point in the ordered phase may be reached either by cooling in a finite field (gray) or by cooling in zero field and then turning on the field (green). For small random fields, the phase diagram for one random-field magnet has been mapped out in detail by V. Jaccarino, A. R. King and D. P. Belanger, *J. Appl. Phys.*, **57**(1) 3291 (1985).

Metastability and Equilibrium States

A system is said to be in a metastable state when its properties are virtually independent of time over a wide range of time scales but ultimately change at some even longer time. In pure systems such behavior occurs near a first-order phase transition. If, for example, a small negative magnetic field is applied to a pure Ising ferromagnet in the ordered state with a positive, or "up," magnetization, a very long time is required to nucleate (critical size) droplets of the "down" state that grow with time and thus reverse the magnetization. During this time, the up state is metastable. There is a large separation between the time scale for the microscopic relaxation that occurs immediately after the field is applied, and the time scale for nucleation of the down state. (See the figure at right.) Such a wide separation of time scales is needed for metastability to be a useful concept.

Distinguishing between metastable states and equilibrium states, which are stable for infinitely long times, is a subtle but important problem related to understanding ground states of an infinite system. For a finite classical system, the ground state (or states, if there is degeneracy) is simply the configuration of lowest energy. However, for infinite systems, especially in the presence of randomness, such a statement is not meaningful! A ground state of an infinite magnet is any configuration whose energy cannot be lowered by changing the spin configuration in *any finite* region. Thus in an Ising ferrmomagnet, the configuration with a single planar domain wall separating up spins from down spins is a truly *stable* ground state, rather than a metastable one, even though it has a higher energy locally than the wholly up configuration does. Bona fide equilibrium domain-wall states can also exist at nonzero temperatures. A recent, somewhat surprising theoretical prediction is that no equilibrium domain-wall–like states exist in spin glasses.[19]



In random Ising magnets domain walls are effectively frozen over long times, local fluctuations being virtually time independent. These domain walls thus produce a kind of metastability. In any decade, or "epoch," of time, however, there is always relaxational activity taking place in some region of a random magnet. Macroscopic properties of the system thus evolve in every epoch, and the clean separation of time scales found in pure systems does not occur. The figure shows the time dependence of a macroscopic variable in a conventional metastable system (blue) and that in a random magnet (red). To distinguish between these two kinds of metastable behavior one must probe a system on a wide range of time scales.

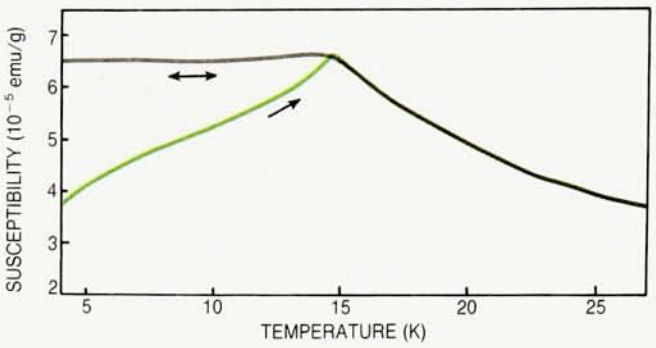
The spectrum of time scales in random magnets is so broad that in any epoch almost all processes in the system are either in nearly perfect equilibrium or completely out of equilibrium, with only a few processes actually equilibrating in that epoch. Ideas based on this partial separation of time scales have led to insights into some experiments on hysteresis and remanence in spin glasses.[19]

magnetization produced by an applied magnetic field H) turns out to be closely related to χ_{SG} in the paramagnetic phase, and so should diverge at the T_c if the spin glass transition is continuous. Measurements of this quantity in several spin glass systems, notably CuMn and AgMn, show strong evidence of such a divergence, and hence of a spin glass transition (see the lower figure on page 64).[5,14] One can never in practice see an actual divergence in a finite system, so the case is not airtight, but combined with other evidence from studies of dynamical effects, the data strongly suggest that at least some real spin glasses do undergo a genuine phase transition in three dimensions.

Equilibrium properties of spin glasses

Having argued that long-range spin glass order occurs in three dimensions, in both simple models and real systems, we now discuss the equilibrium properties of the spin glass phase itself. Much of the theoretical work on spin glasses has been based on an infinite-range model introduced by D. Sherrington and S. Kirkpatrick.[15] The model is defined by equation 1, except that a random exchange bond J_{ij} connects every pair of sites (i, j), not just the nearest-neighbor pairs. Such infinite-range models provide a formulation of mean-field theory, which, at least for pure systems, can be solved exactly (see the box on page 67). Mean-field theory, the first refuge of statistical mechanicians, has been a source of insight into the ordered phases and phase transitions of many complex systems.

The SK model is an ordinary paramagnet at high temperatures. As the critical temperature T_c is approached from above, the spin glass susceptibility χ_{SG} diverges like $(T - T_c)^{-1}$, signaling a continuous spin glass transition. The ferromagnetic susceptibility has a cusp at T_c and remains constant for all $T \leqslant T_c$, while the Edwards–Anderson order parameter grows linearly from zero for T slightly less than T_c. All of this is in accord with the original Edwards–Anderson calculations. The spin glass phase, on the other hand, has been argued by G. Parisi and others[5,16] to have a surprisingly rich structure: For all $T < T_c$ there exist (in a sense that is not yet entirely clear) an infinite number of distinct equilibrium states, or valleys in the free-energy landscape, into which the system may fall when cooled below T_c, much as an Ising ferromagnet spontaneously selects one of the two equivalent states ("up" or "down") available to it below the Curie temperature. Each such state is separated by infinite energy barriers from all the others; having selected one of them, the system remains in it forever. The values of physical quantities such as the energy and Edwards–Anderson order parameter are independent of the sample (that is, of the particular realization of the J_{ij}'s) and of the state. Remarkably, however, some macroscopic quantities depend on the sample even for arbitrarily large systems![5] The spin-glass susceptibility, which diverges as $T \to T_c$ from above, stays infinite for all temperatures below T_c, indicating that the states are extremely sensitive to an



Static susceptibility of the spin glass CuMn for 2.02 atomic percentage of Mn. Below $T_f \approx 15$ K, the measured susceptibility (M/H) is larger when the sample is cooled in a small magnetic field (gray) than when it is cooled in zero field (green). Arrows indicate the direction of temperature changes in the course of the two measurements. Such hysteretic behavior is characteristic of spin glasses. [Adapted from S. Nagata, P. H. Keesom and H. R. Harrison, *Phys. Rev.* B **19**, 1633 (1979).]

applied field. Perhaps most remarkably, the spin glass transition in the SK model persists even in the presence of a magnetic field H, with χ_{SG} diverging at a finite temperature $T_c(H)$ despite the fact that the field explicitly breaks the spin-flip symmetry of the Hamiltonian.

As yet, the relevance of the SK model to the behavior of spin glasses with more realistic short-range interactions remains unclear (see the box on page 67). Building on earlier attempts to calculate the stiffness of short-range spin glasses,[17,18] Fisher and D. A. Huse have proposed a very different picture of the spin glass phase.[19] They hypothesize that like the ordered phases of simpler systems, the spin glass phase can be characterized by its stiffness, which they assume grows with length scale L like L^{θ}. The argument that ordered phases are stable only when the stiffness exponent is positive requires, given the numerical results for spin glasses, that $\theta > 0$ for $d = 3$ and $\theta < 0$ for $d = 2$ or 1. The lowest-energy droplet excitations away from the ground state, which contain L^d spins, typically have free energies of order $F_D(L) \approx \Upsilon L^{\theta}$, where Υ is the (temperature dependent) stiffness modulus of the spin glass. Although such droplets are compact, their diameters being typically of order L, their boundaries can be shown to be *fractal*,[19] with surface areas proportional to L^{d_s}, where $d - 1 < d_s < d$. (See the article by Po-zen Wong on page 24.)

Because of the strong frustration, large sections of the walls separating droplets actually have negative energy (though the total energy must of course be positive). This suggests that droplet energies are roughly given by a sum of terms random in both magnitude and sign, producing an exponent θ much smaller than for ferromagnets, where $\theta = d - 1$. A more careful argument yields the bound $\theta \leqslant (d-1)/2$; the best numerical estimate[17,18] gives $\theta \sim 0.2$ for $d = 3$. The frustration also produces droplets of energies arbitrarily close to zero; even for large L the distribution of droplet energies has a nonzero weight down to $F_D(L) = 0$. Clearly only droplets with free energies $\lesssim T$ will be excited at low temperatures; these droplets are the "active" excitations of the system. Although arbitrarily large droplets can be thermally active at very low temperatures, large active droplets are rare and so tend to be far apart (see the figure on page 65).

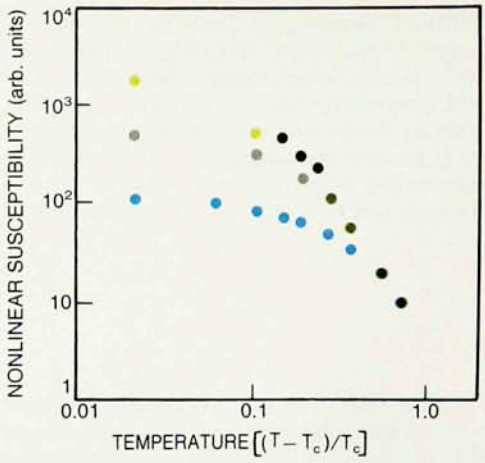
This general picture yields many specific predictions for the properties of the spin glass phase. For example, the connected correlations $C(r)$ between a typical pair of spins separated by a distance r decay exponentially with r. In the rare event that the two spins happen to be part of a large active droplet, however, their correlations are much stronger. The mean-square correlation function—an average of $(C(r))^2$ over all pairs of spins separated by distance r—reflects the strong correlation of such unusual pairs and decays only *algebraically* with r.[19] In consequence, spin glass and nonlinear susceptibilities are infinite at all temperatures in the spin glass phase. The phenomenon of physical quantities (in this case, χ_{SG}, for example) being controlled by rare, statistically unlikely regions of the sample is a subtle but important effect that occurs frequently in disordered systems.

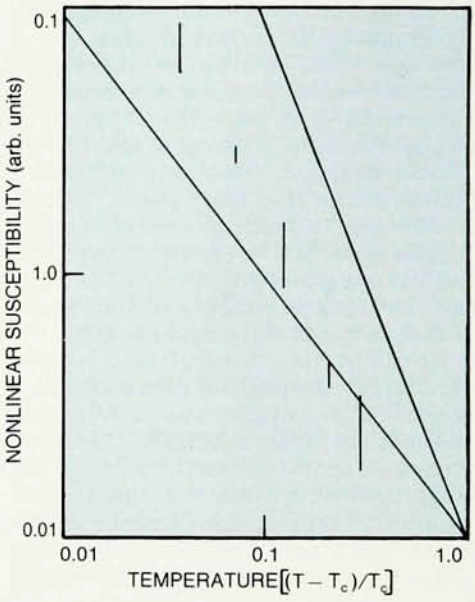
Another interesting prediction of the droplet picture of the spin glass phase is the extreme sensitivity of the equilibrium states to changes in temperature. Arbitrarily small temperature changes upset the delicate balance that produced the equilibrium state at the original temperature, and so result in the reorientation of large regions of the system. As a result, the relative orientations, in equilibrium, of spins located sufficiently far apart change randomly with arbitrarily small changes in temperature! The evolution of the spin glass phase with temperature may therefore be thought of as an infinite sequence of infinitesimal first-order transitions. As we shall see, this phenomenon plays an essential role in preventing spin glasses from reaching equilibrium.

Identification of large-scale, low-energy-cost (active) droplets as the dominant excitations of the spin glass phase also gives rise to some rather striking predictions about the equilibrium dynamics. For example, the power spectrum of the equilibrium magnetization noise is predicted to behave, aside from logarithmic corrections, like $1/\omega$—that is, to exhibit *equilibrium* "$1/f$" noise at low frequencies ω. The magnetic susceptibility, a related quantity, is argued to vary only logarithmically with frequency.[19] Both predictions are in quantitative agreement with recent experiments.[20]

Although some aspects of this emerging picture of the spin glass phase in systems with short-range interactions—for example, the fact that χ_{SG} is infinite—are similar to predictions for the SK model, crucial differences do exist: First, scaling arguments and the intuition gleaned from the study of random-field and random-interface models strongly suggest that Ising-spin glass models with short-range interactions have only one pair of ground states (and hence only *two* equilibrium states for $T > 0$), related to each other by the global spin-flip symmetry![19] The basic physics underlying this assertion is that a system stiff enough to support an ordered spin glass phase is also stiff enough to resist, in the thermodynamic limit, the overturning of any infinite proper subset of the spins. Secondly, the Imry–Ma domain-wall argument[7] (first used in this context by W. L. McMillan[17]) can be used to show that a magnetic field *destroys the spin glass phase*. The reason for this is that $\theta \leqslant (d-1)/2$, so the field energy $\sim HL^{d/2}$ gained by overturning a droplet always dominates the exchange cost of $\sim L^{\theta}$ at large L.

These predictions directly contradict two of the most dramatic features of the spin glass phase suggested by the SK-model results. While there are various possible holes in the arguments leading to these predictions—which are certainly controversial—it is fair to say that no consistent picture of finite-range spin glasses incorporating the SK-model result of many equilibrium states has yet been constructed.





Nonlinear susceptibilities. Top right: Result of Monte Carlo simulations of three-dimensional Ising spin glasses with 8^3 (blue), 16^3 (gray), 32^3 (green) and 64^3 (black) spins, performed on a special purpose computer. (Figure adapted from A. T. Ogielski, *Phys. Rev.* B **32**, 7384 (1988).) Bottom right: Experimental result for the spin glass AgMn with 10.6 atomic percentage of Mn. Straight lines indicate the behavior expected if the nonlinear susceptibility diverges at a critical temperature T_c like $(T-T_c)^{-\gamma}$ with $\gamma=1$ or 2. These data are strong evidence for the existence of a spin glass phase. [Figure adapted from P. Monod and H. Bouchiat, *J. Phys. (Paris) Lett.* **43**, 145 (1982).]

Nonequilibrium phenomena

In comparing with experiment the results of calculations for static, equilibrium properties one implicitly assumes that the system being investigated achieves equilibrium on time scales short compared with those of the experimental probes. This assumption is almost never violated in typical pure systems, but its conspicuous failure in disordered systems has dramatic consequences for the experimental phenomenology.

The degree to which static equilibrium theories accurately describe measurements depends on the rate at which the system equilibrates following a change in some external parameter such as temperature, relative to the rate at which that parameter is varied. Since the equilibrium correlation length ξ diverges at a continuous phase transition, and since correlations cannot grow to infinity in a finite time, any large enough system cooled through a critical point at a finite rate will drop out of equilibrium. (The divergence of ξ in the vicinity of a critical point is always accompanied by the growth of the characteristic relaxation, or equilibration, time, since many spins must relax coherently.[21] This phenomenon is called "critical slowing down.") For typical pure systems, however, the equilibration rate is so rapid that even macroscopic systems can equilibrate in reasonable experimental times. For example, the characteristic time τ required for a pure Ising system at its critical point to equilibrate on a length scale L is $\tau\sim\tau_0(L/a)^z$, where a, a typical lattice spacing, is a few angstroms, $\tau_0\sim10^{-11}$ sec and the exponent $z\sim2$. Hence in 1 second the system will relax over distances on the order of $\sim10^6$ Å—that is, over a macroscopic length scale. Thus the immense speed of microscopic processes allows pure systems to equilibrate quickly even near critical points. For most purposes, therefore, one can ignore nonequilibrium effects.

One can of course force the system out of equilibrium by increasing the rate at which it is perturbed. Let us, for example, consider quenching a system instantaneously from a high temperature, where ξ is small, to a temperature below T_c and study the "domain growth kinetics"—the rate of growth of the initially small clusters of up and down spins. In pure Ising-like systems the radius $R(t)$ of typical domains of correlated spins follows the Lifshitz growth law, $R(t)\sim t^{1/2}$. This is a consequence of the growth's being controlled by surface tension[22]: The rate dR/dt at which small droplets shrink and large ones grow is proportional to the droplets' curvature, $1/R$.

In random systems, by contrast, the droplet boundaries tend to get pinned by impurities, which retards the growth. This is readily seen in the context of the random-exchange model,[23] though similar considerations apply equally well to random-field magnets and to spin glasses: The domain walls separating up and down spins in random-exchange systems minimize their energy cost by passing between sites connected by weak bonds (see the figure on page 57), producing wall meandering, or roughening. The walls must therefore surmount the energy barriers presented by nearby stronger bonds in order to move. The possibility of finding weak bonds by meandering increases with L, and hence so do the energy barriers. One expects the energy barriers opposing the equilibration of random systems of length scale L to scale like $B(L)\sim L^{\psi}$ for large L. This result was first derived by J. Villain and by Grinstein and J. F. Fernandez for random-field magnets. The exponent ψ depends on the dimension d and the type of disorder. Since the typical time required to surmount a barrier B at temperature T is proportional to the Arrhenius factor $\exp(B/T)$, the algebraic form for $B(L)$ immediately implies that the time required for equilibration on length scale L in the ordered phase of a random system is $\tau(L)\sim\exp(L^{\psi}/T)$. Conversely, the length scale $R(t)$ on which the system can equilibrate in time t or, equivalently, the maximum length to which correlations will extend following a quench into the ordered phase is

$$R(t)\sim(\ln t)^{1/\psi}$$

This remarkably slow growth of ordered domains underlies the hysteresis observed so ubiquitously in random systems. For ψ's of order unity, which is typically the case, domain walls are effectively frozen; many orders of

magnitude of increase in t are required to produce modest changes in the correlation length and other measurable quantities. Let us now discuss qualitatively some of the implications of this slow domain growth in each of the classes of random magnets:

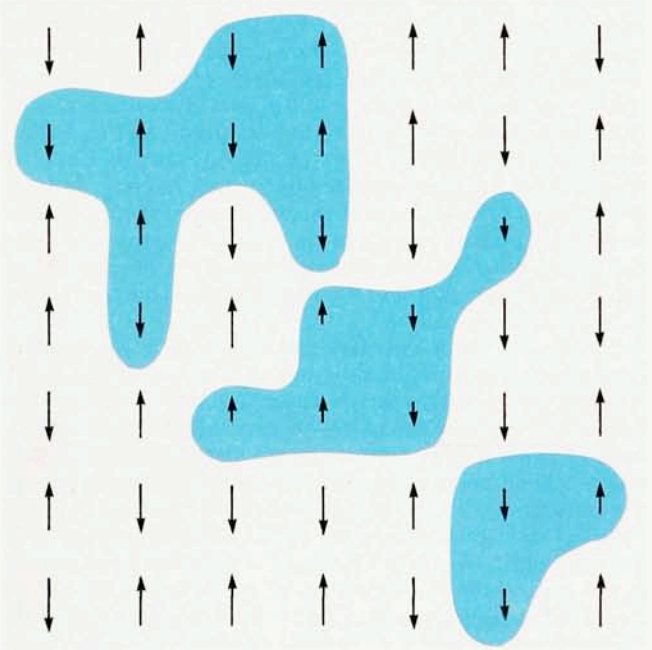
Random-exchange systems ought to exhibit logarithmically slow growth of correlations following a rapid quench to a temperature below T_c for both $d=2$ and $d=3$. But under slow cooling toward T_c the divergence of correlation times τ in the critical region is believed to be governed by conventional critical slowing down with $\tau \sim \xi^z$, just as in pure systems. Unless z is anomalously large, such algebraic slowing down allows, as argued above, equilibration on macroscopic length scales in ordinary experimental times. Hence under normal, gradual cooling toward T_c correlation lengths can readily become quite large, and remain large for $T < T_c$, effectively masking the domain-wall freezing and concomitant slow growth expected at lower temperatures.

Random fields. Arguments about energy barriers at zero temperature[23] suggest that the exponent ψ is unity for both $d=2$ and $d=3$. So the slow equilibration following quenches to low T is qualitatively similar to that of random-exchange magnets. The situation under slow cooling is quite different, however. We remarked earlier that the fluctuations due to the random fields dominate those due to thermal effects. Remarkably, this remains true even in the critical region, not just at low temperatures. Thus the characteristic free-energy scale, and hence the height of typical energy barriers, is set not by thermal energies but by the random fields. Since the equilibrium correlation length ξ is the only important length in the problem near the critical point, it is reasonable to assume that the barriers due to random fields scale algebraically with this length: $B \sim \xi^{\psi_c}$, where ψ_c is a critical exponent of order unity. It follows immediately that the characteristic relaxation time τ diverges exponentially with ξ, $\tau \sim \exp(\xi^{\psi_c}/T)$, as one approaches T_c from above.[23,24] This phenomenon, known as activated dynamic scaling, means that as the temperature is lowered τ will increase beyond typical measuring times at a temperature somewhat greater than T_c. At that point the system will fall rather abruptly out of equilibrium, and ξ will remain essentially fixed at a finite value as one cools further. This corresponds closely to what is observed experimentally in random-field systems when a sample is cooled in a magnetic field (see figures on pages 60 and 61). The value at which ξ freezes can be quite small (~ 100 Å, say) even for modest fields.

Thus there is a rather sharply defined "irreversibility" curve in the random-field–temperature plane at which the system effectively freezes, even under slow cooling, subsequent growth of correlations in time proceeding logarithmically slowly. (The position of this freezing line has now been mapped out rather precisely in experiments; see the figure on page 61.) One simply cannot probe the true equilibrium behavior of random-field magnets on any reasonable experimental time scale; nonequilibrium dynamics play the dominant role. Note that samples cooled below T_c in zero field presumably freeze too: They cannot equilibrate in typical experimental times when the field is applied. In this case, however, the frozen state does have long-range order, and so is "closer" to the equilibrium state of the three-dimensional random-field model than is the state achieved under field cooling.

Spin glasses. The sharpness of the susceptibility cusp and the apparent divergence of the nonlinear susceptibility, found in both numerical simulations and some experiments, show that spin glass correlations become large as the temperature is lowered to T_c at $H=0$. This is consistent with the relaxation times diverging only *algebraically* at T_c (as in the random exchange model), a behavior suggested by theoretical arguments. (At present, however, the data are also consistent with activated dynamic scaling, such as occurs in the random-field magnets.) Unlike in either random-exchange or random-field magnets, however, the long correlations achieved in spin glasses during slow cooling to T_c are not retained as T is lowered below T_c. This is a consequence of the sensitivity of the spin glass state to temperature changes.[19] As T is lowered below T_c, arbitrarily large droplets of spins must be flipped for the system to be in equilibrium at each new temperature. This process requires overcoming the barriers, $B(L) \sim L^{\psi}$, and so proceeds logarithmically slowly. Hence the system, which is essentially frozen close to the configuration attained near T_c, drops further and further out of equilibrium as T decreases.

While the existence of a true phase transition in spin glasses in a nonzero magnetic field remains controversial, one expects the system effectively to freeze at a rather well-defined curve, $T = T_f(H)$, which passes just above the zero-field critical point in the (T,H) plane. The freezing will occur whether or not there is an equilibrium ordered phase for nonzero field. If, as in the infinite-range model, a finite-field transition does occur, then freezing will set in just above the thermodynamic phase boundary, as just discussed for $H=0$. If, on the other hand, the droplet scaling picture is correct and there is no transition in a field, τ will not diverge for $H \neq 0$ but nonetheless will get



Excitations in the ordered phase of an Ising spin glass consist of active droplets of spins (blue), which fluctuate nearly independently, so the single-site magnetization (here indicated by the length of the arrows) is almost the same for every site in a droplet but is reduced compared with its value for spins that do not belong to any active droplet. Active droplets of all sizes are possible, but small dropletes are more likely than big ones. The shape and location of large active droplets changes markedly with infinitesimal changes in the temperature.

extremely large in the vicinity of the zero-field critical point at $H = 0$, $T = T_c$. Thus there will still be a sharply-defined line through this point at which τ will exceed typical experimental times and the system will effectively freeze. Aside from the weak (logarithmic) dependence of the position of the freezing line on the time scale of the experiment in this latter scenario, the dependence of $T_f(H)$ on H asymptotically close to $H = 0$ should be independent of the occurrence of a true transition in equilibrium. As in random-field magnets, therefore, one finds that the dynamics control the experimentally accessible phenomena and the thermodynamic behavior plays only a supporting role.

Summary and outlook

We have seen that strongly competing interactions in random magnetic systems produce new equilibrium ordered phases without counterparts in pure systems. Perhaps more striking is the dominant role played by the dynamics: Even on macroscopic time scales random systems below their critical points are often far from equilibrium; only by understanding their nonequilibrium behavior can one hope to explain the experimental observations.

For random-field magnets, a qualitative understanding of the equilibrium phase diagram, critical behavior and experimental history dependence has been achieved, though quantitative agreement between theory and experiment has yet to be attained.

The situation for spin glasses is far less clear. While both model Ising-spin glasses in three dimensions and some three-dimensional experimental systems apparently undergo equilibrium phase transitions, the experiments typically are performed on systems with Heisenberg spins. The best theoretical estimates for these systems suggest that $d_l > 3$, and hence that a spin glass phase *does not exist* in three dimensions.[5] This remains a puzzle. The effect of the long-range RKKY interactions present in metallic spin glass systems is likewise poorly understood.[5] Indeed, whether all or only some of the experimental spin glasses have true phase transitions is far from clear. It seems likely that many of the quintessential spin glass properties can occur in systems that do not undergo a thermodynamic phase transition.

Even for three-dimensional Ising-spin glass models, crucial equilibrium issues such as the stability of the ordered phase with respect to an applied field and the number of equilibrium states in zero field remain controversial. Given the difficulties of disentangling a true phase transition from a nonequilibrium freezing line and the absence of a good experimental Ising-spin glass, prospects for settling the former issue experimentally are not too bright. As for the number of equilibrium states, it is not clear how the existence of many states would be manifest experimentally. There has nonetheless been encouraging agreement between theory and experiment for certain *nonequilibrium* phenomena (see the box on page 62). While many of the complicated dynamical and hysteretic phenomena observed in spin glasses over the last 15 years remain unexplained, progress in the understanding of important low-energy excitations makes one optimistic that theoretical and experimental studies of the spin glass phase will continue to converge.

The principle of universality (see the box on the opposite page) suggests that concepts recently developed for random magnets should have wide applicability to other quenched random systems. For example, ideas about random-field magnets have already been used to study phase separation of fluids in porous media and wetting on random substrates. The understanding that certain macroscopic properties are governed by rare regions of a sample has been applied to quantum problems such as the onset of superfluidity in dirty superconductors and superfluids at very low temperature. Spin glass notions have found their way into neurobiology (see the article by Haim Sompolinsky on page 70), molecular biology, theories of evolution, and a host of optimization problems such as designing computer chips, finding optimum paths for a travelling salesman and partitioning graphs under specific constraints. Some of these problems have effective interactions that are long ranged, so methods developed for the Sherrington–Kirkpatrick model are directly applicable. The "simulated annealing" approach to optimization, in which the extremal values of complicated cost functions of many variables are obtained by what amounts to a series of Monte Carlo simulations at successively lower temperatures, had its genesis in numerical attempts to find the ground state of Ising-spin glasses.

There is reason to hope that the mechanisms of dynamical freezing found in systems with quenched disorder will prove useful in systems such as real glasses, which apparently generate their own effective randomness. It is worth mentioning in this connection the recent interest in systems that exhibit dynamical phase transitions without an underlying transition in the equilibrium properties. The program of analyzing ordered phases in terms of stiffness and droplet excitations, the power of which has been so compellingly demonstrated in the context of random magnets, turns out to generalize rather nicely to complicated dynamical systems without an underlying Hamiltonian or energy, such as cellular automata. This approach is already being fruitfully employed in the elucidation of the phases and phase transitions of these systems.

Theoretical investigations of random magnets began during the 1970s with attempts to understand the equilibrium phase transitions and critical phenomena via techniques, such as formal ϵ expansions, borrowed from the study of pure magnets. These tools have proved enormously successful for pure systems, whose ordered phases and dynamics are relatively straightforward, but their application to random magnets has produced many anomalies and misleading results. It has gradually become clear that such tools are simply inadequate for random systems, whose ordered phases can be intriguingly complex and in which subtle interplay between nonequilibrium dynamic phenomena and the underlying equilibrium behavior is observed.

Though new tools designed specifically for random systems are, with a few exceptions, still quite crude, they have led to considerable qualitative understanding. Further refinements are certainly needed. These are likely to require the grand scheme of Wilson's renormalization group if they are to capture the many time and length scales and the statistical variations that contribute to the behavior of disordered systems. The field of magnetism has finally reached maturity, however: The basic issues and questions are now clear. Research in this fascinating area is at last beginning to wrest some order from the jaws of disorder.

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Renormalization Group Techniques in Random Systems

The revolution of the 1970s in our understanding of the critical phenomena at continuous phase transitions was achieved with the concepts and language of the renormalization group. The essential idea is to focus on the behavior of a system at successively longer length scales. Universality—the remarkable property that the behavior of a macroscopic system at large length scales is independent of microscopic details—is a consequence of the convergence of the effective Hamiltonians that describe the system at different scales to one of several fixed points as the length scale is increased. Progress in calculating critical exponents using these ideas was triggered by the realization that mean-field theory provides a base about which systematic perturbative renormalization group expansions could be developed. For pure systems, mean-field theory is exact in the limit that the interactions become infinite ranged, and captures much of the qualitative physics of systems with short-range interactions. It even yields the correct critical exponents for all dimensions d above the so-called upper critical dimension, d_u, of the system. The renormalizaton group enabled one to calculate the exponents for $d < d_u$ in an expansion in the small parameter $\epsilon \equiv d_u - d$, with d being treated as a continuous variable. For pure magnets, such as those described by the Ising model, for which d_u is 4, these ϵ-expansion methods give accurate exponent values even for $d = 3$.

It was therefore natural to construct mean-field theories for random systems and then to attempt expansions about some appropriate upper critical dimension. This program worked well for random-exchange magnets, but despite the expenditure of enormous effort, it has so far failed to provide much insight into the subtleties of the random-field and spin glass models with realistic (short-range) interactions. Instead, the approach has produced some rather grave misapprehensions. Consider, for example, the random-field Ising model, for which the mean-field theory is rather straightforward. From formal ϵ expansions around the upper critical dimension, which is 6 for this model, it was concluded that the critical exponents at the ferromagnetic phase transition in any dimension d were the same as those for the pure Ising model in $d - 2$ dimensions. From this result, which is known as "dimensional reduction," follows the prediction that the lower critical dimension of the random-field model is 3. This prediction is now known to be incorrect. Moreover, the ϵ expansion misses completely the crucial slow dynamics and the possibility of history-dependent effects in the model. Both of these failures arise from a difficulty encountered frequently when applying formal perturbative methods to random systems: In these methods the procedures for averaging over randomness typically neglect the effects of statistically unlikely regions of the random system; such regions can make essential contributions to physical quantities.

For spin glasses, much of the theoretical effort has been expended on the infinite-range Sherrington–Kirkpatrick model, yielding many interesting results and also insights into difficult optimization problems. But so far there is no clear indication that the behavior of this model is relevant to short-range spin glasses. Attempts to expand perturbatively about it continue to be plagued with severe technical and conceptual difficulties. Indeed, it is still unclear that the SK model really represents a high-dimensional limit of short-range spin glasses.

In spite of these difficulties in obtaining quantitative results, *qualitative* renormalization group ideas underlie much of the progress toward understanding random magnets that we have sketched in this article.

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