QM417 Midterm Fall 2010

Name (First, and Last):

Useful formula

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-), \quad p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-), \quad a_\pm = \sqrt{\frac{1}{2\hbar m\omega}}(\mp ip + m\omega x),$$

$$a_+\psi_n = \sqrt{n+1}\psi_{n+1}, \qquad a_-\psi_n = \sqrt{n}\psi_{n-1}, \qquad [a_-, a_+] = 1$$

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1. (2pts) True or False: In the absence of external forces, electrons move along sinusoidal paths.
   A. True
   B. False

2. (2pts) Consider an electron with the potential energy $U(x) = \begin{cases} 0 & 0 < x < L \\ \infty & x < 0 \quad or \quad x > L \end{cases}$.

   Your electron is in the lowest energy state of this potential energy, with a wave function $\psi(x) = \psi_1(x)$ and a corresponding energy E_1. Suppose you first measure the position of this electron very precisely, without destroying the electron. *After* measuring the position, you measure the *energy* of the same electron. Which of the following statements describes the result of this energy measurement?

   A. The value that you measure will be E_1.
   B. The value that you measure could possibly be E_1.
   C. The value that you measure will *not* be E_1.

3. (4pts) A theorist has constructed a model Hamiltonian as $H = \epsilon(|1\rangle\langle 2| - |2\rangle\langle 1| + b|2\rangle\langle 2|)$, where "$\epsilon$" is a non-zero real constant and "b" is another constant to be determined. In order to make this a valid Hamiltonian, what is the requirement for "b"?
   A. "b" should be imaginary
   B. "b" should be real
   C. "b" should be zero
   D. No choice of "b" will make this a valid Hamiltonian

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4. Suppose that a particle starts out in a linear combination of two normalized stationary states, $\psi_0(x)$ and $\psi_1(x)$, which are the ground state and the first excited state with the corresponding eigen-energies of E_0 and E_1, respectively:

$$\Psi(x,0) = A[2\psi_0(x) + \psi_1(x)]$$

Here, also assume that $\psi_0(x)$ and $\psi_1(x)$ are both real and that there exist infinite number of other stationary states in addition to these two states.

(a) (2pts) Determine the normalization constant, *A.*

(b) (2pts) Does $\Psi(x,0)$ satisfy the time independent Schroedinger's equation?

(c) (4pts) Find the wave function $\Psi(x,t)$.

(d) (2pts) Does $\Psi(x,t)$ satisfy the time dependent Schroedinger's equation?

(e) (2pts) When you measure the energy of the system at *t*, what is the probability of observing energy value of $(E_0 + E_1)/2$?

(f) (2pts) What is the expectation value of the Hamiltonian at *t*?

(g) (2pts) When you measure the energy of the system at t, what is the probability of observing the energy value of E_0?

(h) (2pts) Now, assume that the energy measurement yielded E_0at a particular time (we reset our clock to time zero at this time), if you call the state immediately after this measurement, $\Omega(x,0)$, what is $\Omega(x,0)$ in terms of $\psi_0(x)$ and $\psi_1(x)$?

5. The harmonic oscillator is described by the Hamiltonian, $H = \frac{1}{2m}[p^2 + (m\omega x)^2]$.

(a) (8pts) Find $\langle p \rangle$ and $\langle p^2 \rangle$ in the *n*th state of the harmonic oscillator.

(b) (6pts) Starting from the ground state (n=0) eigen-function, $\psi_0(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}$, where *A* is a normalization constant, find the normalized n=1 eigen-function ($\psi_1(x)$) by applying the ladder operator given above. You can keep the constant *A* in your final answer.

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6. (4pts) Show whether the operator, $i\frac{d}{dx}$, is hermitian or not.

7. The Hamiltonian of a two-level system is given by $H = \epsilon(|1\rangle\langle 2| + |2\rangle\langle 1|)$, where ϵ is a non-zero real constant, and $|1\rangle$ and $|2\rangle$ form an orthonormal basis.

   (a) (2pts) With $|1\rangle$ and $|2\rangle$ as the basis, find the matrix representation of *H.*

   (b) (6pts) Find eigenenergies, and normalized eigenstates as linear combinations of $|1\rangle$ and $|2\rangle$.

   (c) (8pts) If the system starts out (at *t*=0) in state $|1\rangle$, what is the probability of finding it in state $|1\rangle$ at a later time *t*?

8. (8 pts) Consider a particle in an infinite well shown below. Without solving the Schroedinger equation, sketch the probability density $|\psi(x)|^2$ of the eigenfunctions corresponding to the ground state (E_0) and the second excited state (E_2), and explain all the key qualitative features.