QM417 Midterm Fall 2009

Name (First, and Last):

Useful formula

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-), \quad p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-), \quad a_\pm = \sqrt{\frac{1}{2\hbar m\omega}}(\mp ip + m\omega x),$$

$$a_+\psi_n = \sqrt{n+1}\psi_{n+1}, \qquad a_-\psi_n = \sqrt{n}\psi_{n-1}, \qquad [a_-, a_+] = 1$$

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1. Suppose that a particle starts out in a linear combination of two normalized stationary states, $\psi_0(x)$ and $\psi_1(x)$, which are the ground state and the first excited state with the corresponding eigen-energies of E_0 and E_1, respectively:

$$\Psi(x,0) = A[\psi_0(x) + \psi_1(x)]$$

Here, also assume that $\psi_0(x)$ and $\psi_1(x)$ are both real and that there exist infinite number of other stationary states in addition to these two states.

(a) (2pts) Determine the normalization constant, *A*.

(b) (2pts) Does $\Psi(x,0)$ satisfy the time independent Schroedinger's equation?

(c) (4pts) Find the wave function $\Psi(x,t)$.

(d) (8pts) Evaluate $|\Psi(0,t)|^2$, and sketch its shape as a function of time. What are its maximum and minimum values if both $\psi_0(0)$ and $\psi_1(0)$ are positive.

(e) (2pts) Does $\Psi(x,t)$ satisfy the time dependent Schroedinger's equation?

(f) (2pts) When you measure the energy of the system at *t*, what is the probability of observing energy value of $(E_0 + E_1)/2$?

(g) (2pts) What is the expectation value of the Hamiltonian at *t*?

(h) (2pts) When you measure the energy of the system at t, what is the probability of observing the energy value of E_1?

(i) (2pts) Now, assume that the energy measurement yielded E_1 at a particular time (we reset our clock to time zero at this time), if you call the state immediately after this measurement, $\Omega(x,0)$, what is $\Omega(x,0)$ in terms of $\psi_0(x)$ and $\psi_1(x)$?

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(j) (2pts) Now immediately after (i), if you measure the position of the particle represented by the wave function, which of the following statements is correct about the state of the system right after the position measurement?

① The state should be $\psi_0(x)$.
② The state should be $\psi_1(x)$.
③ The state should be either $\psi_0(x)$ or $\psi_1(x)$, but we do not know which one of these two.
④ The state should be some linear combination of $\psi_0(x)$ and $\psi_1(x)$, but we do not know the exact coefficient for each term.
⑤ The state can be any one of the energy eigenstates, $\psi_0(x), \psi_1(x), \psi_2(x), \ldots, \psi_n(x)$, but we do not know which one it is.
⑥ None of the above statements are correct.

(k) (2pts) Now immediately after the position measurement as described in (j), if you measure the energy of the system, which one of the following statements is correct about the measured energy value?
① It should be E_0.
② It should be E_1.
③ It should b either E_0 or E_1.
④ It should be some value between E_0 and E_1.
⑤ It should be one of E_0, E_1, E_2,…, E_n,…
⑥ It can be any real number equal or larger than E_0.
⑦ It can be any real number.
⑧ None of the above statements are correct.

2. The harmonic oscillator is described by the Hamiltonian, $H = \frac{1}{2m}[p^2 + (m\omega x)^2]$.
   (a) (8pts) Find $\langle x^2 \rangle$ in the *n*th state of the harmonic oscillator.

   (b) (4pts) Starting from the ground state (n=0) wave function, $\psi_0(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}$, where *A* is the normalization constant, find the normalized n=1 wave function ($\psi_1(x)$) by applying the ladder operator. You can keep the constant *A* in your final answer.

3. (4pts) Show whether the operator, $\frac{d}{dx}$, is hermitian or not.

4. (4pts) What is the hermitian conjugate of $\begin{pmatrix} 1 & 1+i \\ 2 & 2i \end{pmatrix}$

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5. (4pts) What is the hermitian conjugate of $\hat{x}\hat{p}$, the product of the position and the momentum operator?

6. The Hamiltonian of a two-level system is given by $H = \epsilon(|1\rangle\langle 1| + |2\rangle\langle 2| - |1\rangle\langle 2| - |2\rangle\langle 1|)$, where $|1\rangle$ and $|2\rangle$ form an orthonormal basis.

   (a) (2pts) With $|1\rangle = \binom{1}{0}$ and $|2\rangle = \binom{0}{1}$ as the basis, find the matrix representation of *H.*

   (b) (6pts) Find eigenenergies, and normalized eigenstates as linear combinations of $|1\rangle$ and $|2\rangle$.

   (c) (8pts) If the system starts out (at *t*=0) in state $|1\rangle$, what is its state at time *t*?