417 Final Exam, Dec. 22 2010

Student Name: First: Last:

Useful formula

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$,

Table 4.2: The first few spherical harmonics, $Y_l^m(\theta, \phi)$.

$Y_0^0 = \left(\frac{1}{4\pi}\right)^{1/2}$	$Y_2^{\pm 2} = \left(\frac{15}{32\pi}\right)^{1/2} \sin^2\theta e^{\pm 2i\phi}$
$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$	$Y_3^0 = \left(\frac{7}{16\pi}\right)^{1/2} (5\cos^3\theta - 3\cos\theta)$
$Y_1^{\pm 1} = \mp\left(\frac{3}{8\pi}\right)^{1/2} \sin\theta e^{\pm i\phi}$	$Y_3^{\pm 1} = \mp\left(\frac{21}{64\pi}\right)^{1/2} \sin\theta(5\cos^2\theta - 1)e^{\pm i\phi}$
$Y_2^0 = \left(\frac{5}{16\pi}\right)^{1/2} (3\cos^2\theta - 1)$	$Y_3^{\pm 2} = \left(\frac{105}{32\pi}\right)^{1/2} \sin^2\theta\cos\theta e^{\pm 2i\phi}$
$Y_2^{\pm 1} = \mp\left(\frac{15}{8\pi}\right)^{1/2} \sin\theta\cos\theta e^{\pm i\phi}$	$Y_3^{\pm 3} = \mp\left(\frac{35}{64\pi}\right)^{1/2} \sin^3\theta e^{\pm 3i\phi}$

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1. (4pts) Can the momentum operator "p" and a Hamiltonian share the same eigenfunction? Explain.

2. (4pts) A theorist has constructed a model Hamiltonian as
$H = \epsilon(i|1\rangle\langle 2| - i|2\rangle\langle 1| - b|2\rangle\langle 2|)$, where "$\epsilon$" is a non-zero real constant and "b" is another constant to be determined. In order to make this a valid Hamiltonian, what is the requirement for "b"?
A. "b" should be imaginary
B. "b" should be real
C. "b" should be zero
D. No choice of "b" will make this a valid Hamiltonian

3. An electron with spin up is in the ψ_{211} state of hydrogen atom.
   (a) (2pts) If you measure L^2 of this electron, where L is the orbital angular momentum operator, what value(s) would you get?

   (b) (4pts) What are the possible values for the total angular momentum quantum number, J, and the total magnetic quantum number, m, of this electron.

4. A spin-1/2 particle at rest in a uniform magnetic field pointing in the z-direction is described by the Hamiltonian:

$$H = -\gamma B_0 S_z.$$

   (a) (2 pts) Write down the matrix describing this Hamiltonian: our basis is the standard $|m_z = \frac{1}{2}\rangle = \binom{1}{0}$ and $|m_z = -\frac{1}{2}\rangle = \binom{0}{1}$.

   (b) (4 pts) If, at t = 0, measurement of S_y resulted in $\frac{\hbar}{2}$, what is the spinor $\chi(t=0)$ of the state right after the measurement? (Do not forget to normalize.)

   (c) (4 pts) At a later time t (>0), what is the corresponding spinor $\chi(t)$?

   (d) (4 pts) If you measure S_z at time t, what are the possible values and what are their probabilities?

5. (8pts) Vanadium has 23 electrons. Using the orbital filling rules and Hund's rules, suggest its most likely electron configuration and provide the angular momentum quantum numbers in terms of the spectroscopic notation $({}^{2S+1}L_J)$.

6. (8pts) Based on the variational principle, find an upper bound on the ground state energy of the one-dimensional infinite square well, $V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & x < 0 \quad or \quad x > a \end{cases}$, using the triangular trial wave function:

$$\psi(x) = \begin{cases} Ax, & \text{if } 0 \le x \le a/2 \\ A(a-x), & \text{if } \frac{a}{2} \le x \le a \\ 0, & \text{otherwise} \end{cases}$$

7. (8pts) Consider a step potential barrier, $V(x) = \begin{cases} V_0 & 0 < x < a \\ 0 & x < 0 \quad or \quad x > a \end{cases}$

When an electron with energy between 0 and V_0 is injected from left toward this step potential barrier, the transmission coefficient was measured as T_0. Now if the width of the step potential is doubled, that is, if 'a' becomes '$2a$', based on the WKB approximation, what is the new transmission coefficient in terms of T_0?

8. (8pts) A hydrogen atom is placed in a time-dependent electric field $\mathbf{E} = E(t)\hat{y}$. According to the first-order time-dependent perturbation theory, this electrical field can cause the electron of the hydrogen atom to make transitions between (otherwise stationary) states. If the electron is initially in its ground state ($|nlm\rangle = |100\rangle$) and if we consider its transition to the quadruply degenerate first excited states ($|nlm\rangle = \{|200\rangle, |21 \pm 1\rangle, |210\rangle\}$), which of these four transitions are forbidden based on the first order time-dependent perturbation theory? Remember that $y = r\sin\theta \sin\phi$ in spherical coordinates.