1. A spin-1/2 particle at rest in a uniform magnetic field pointing in the z-direction is described by the Hamiltonian:

$$H = -\gamma B_0 S_z.$$

(a) (2 pts) Write down the matrix describing this Hamiltonian: our basis is the standard $|m_z = \frac{1}{2}\rangle = \binom{1}{0}$ and $|m_z = -\frac{1}{2}\rangle = \binom{0}{1}$.

(b) (2 pts) If, at t = 0, measurement of S_x resulted in $\frac{\hbar}{2}$, what is the spinor χ(t=0) of the state right after the measurement? (Do not forget to normalize.)

(c) (3 pts) At a later time t (>0), what is the corresponding spinor χ(t)?

(d) (3 pts) Evaluate $\langle S_y \rangle$ at time t.

2. Now in Prob. 1, the magnetic field is replaced by an oscillating field $\mathbf{B} = B_0 \cos(\omega t)\, \hat{k}$
(a) (2pts) Construct the Hamiltonian matrix for this system.

(b) (8pts) The electron starts out (at t=0) in the spin-up state with respect to the x-axis. Determine χ(t) at any subsequent time: Note that now the Hamiltonian is time dependent.

3. (10 pts) An electron with spin down is in the state ψ_{310} of the hydrogen atom. If you could measure the total angular momentum squared of the electron alone (not including the proton spin), what values might you get, and what is the probability of each.

4. Two non-interacting indistinguishable spin ½ electrons are in a 1-D infinite square potential well ($V(x) = 0$ for $0 < x < a$, $V(x) = \infty$, otherwise).
(a) (5pts) What is (are) the spatial wave function(s) of the ground state? Also describe their corresponding spin configurations (singlet or triplet?)

(b) (5pts) What is (are) the spatial wave function(s) of the first excited state? Also describe their corresponding spin configurations (singlet or triplet?)

(c) (5pts) Now, if we take into account coulomb interaction between the electrons as a small perturbation (don't attempt to do any calculation, though), describe how the energy levels of the above states change with their spin configurations into consideration.

5. (10pts) A particle is in the linear potential $V(x) = \alpha|x|$

Use WKB approximation to estimate the ground state energy of this system.

6. Suppose we perturb 2D infinite square potential well (V(x,y) = 0 if 0 < x, y < a, V(x,y)= ∞ otherwise) by putting a delta function "bump" at the point (a/4, 3a/4):

$$H' = a^2 V_0 \delta\left(x - \frac{a}{4}\right)\delta(y - \frac{3a}{4})$$

(a) (5 pts) Write down the wave functions and the energy of the unperturbed first excited states.

(b) (10 pts) Find the first-order corrections to the energy of the doubly degenerate first excited states.

7. (10 pts) A hydrogen atom is placed in a time-dependent electric field $\mathbf{E} = E(t)\hat{\imath}$.

Among the four transitions between the ground state (n=1) and the quadruply degenerate first excited states (n=2), which of these four are forbidden? Remember that $x = r\sin\theta\cos\phi$ in spherical coordinates.