Quantum Mechanics and Atomic Physics (361)

Midterm, Fall 2011

Name (Please write it legibly): ______________________

** This exam is composed of two pages, and so if your second page is missing, let the proctor know immediately **

1. (3pts) Does $\Psi(x,t) = A\sin(x - 2t)$ satisfy the time-dependent Schroedinger equation with a time-independent potential function? If so, what is the energy of this system?

2. (3pts) Does $\Psi(x,t) = Ae^{i(x-2t)}$ satisfy the time-dependent Schroedinger equation with a time-independent potential function? If so, what is the energy of this system?

3. The wavefunction of a particle at a particular time is given by

$$\psi(x) = \begin{cases} A\left(1 + e^{ix}\right), & if \quad -\pi < x < \pi \\ 0, & else \end{cases}.$$

(a) (3pts) Sketch the probability of finding the particle as a function of position.

(b) (3pts) Find the normalization constant A.

(c) (3pts) Find the expectation value of momentum in this state.

4. Suppose that a particle starts out in a linear combination of two normalized states, $\psi_0(x)$ and $\psi_1(x)$, which are the ground state and the first excited state with the corresponding eigen-energies of E_0 and E_1, respectively:

$$\Psi(x,0) = A[2\psi_0(x) + \psi_1(x)]$$

Here, also assume that $\psi_0(x)$ and $\psi_1(x)$ are both real and that there exist many other energy engen-states in addition to these two states.

(a) (3pts) Determine the normalization constant, *A.*

(b) (3pts) What is the expectation value of the Hamiltonian?

(c) (3pts) What is the probability of observing the energy value of E_0?

(d) (3pts) What is the probability of observing energy value of $(E_0 + E_1)/2$?

(e) (3pts) Find the wavefunction $\Psi(x,t)$ at a later time t.

(f) (3pts) Now, assume that the energy measurement yielded E_1 at a particular time (we reset our clock to time zero at this time), if you call the state immediately after this measurement, $\Omega(x,0)$, what is $\Omega(x,0)$ in terms of $\psi_0(x)$ and $\psi_1(x)$?

5. Harmonic oscillator is described by the Hamiltonian, $H = \frac{1}{2m}[p^2 + (m\omega x)^2]$. We found in class that its energy eigenvalues are given by $E_n = \hbar\omega(n + \frac{1}{2})$ with corresponding eigenfunctions, $\psi_n(x) = A_n H_n(\alpha x) e^{-\alpha^2 x^2/2}$, where A_n is a normalization constant and $\alpha \equiv (\frac{m\omega}{\hbar})^{1/2}$.

(a) (3pts) Find the expectation values of momentum and position for $\psi_n(x)$. Hint: think about the even- and oddness of the integrand before you try integration.

(b) (3pts) In your recent homework, using the Virial theorem, you found that <KE> = <PE>= $\frac{\hbar\omega}{2}(n + \frac{1}{2})$ for an energy eigenstate $\psi_n(x)$. Using this result, evaluate the expectation values of p^2 and x^2 for this state. Then combining with the result in (a), evaluate $\Delta x \Delta p$. Does your result satisfy the uncertainty principle?

(c) (3pts) If the harmonic oscillator is in n= 4 state, sketch the probability of finding the particle as a function of position. In your sketch, indicate the location of the classical turning points and x=0 point.