HW #3 Solution

1. Reed, Prob. 3-3

**Problem 3-3**

Using classical arguments, derive an expression for the speed v of a particle in a one-dimensional infinite potential well. Apply your result to the case of an electron in a well with L = 1Å. For what value of n does v exceed the speed of light? To what energy does this correspond?

By equating the energy of a particle in state n of an infinite well to the classical kinetic energy $mv^2/2$,

$$E = \frac{n^2 h^2}{8mL^2} = \frac{mv^2}{2},$$

we find

$$v = \frac{nh}{2mL}.$$

For an electron in a well with L = 1Å, this gives v = $(3.637 \times 10^6\ n)$ m/sec; v exceeds c for n ~ 82, which corresponds to an energy of 259 kV.

2. Reed, Prob. 3-4

**Problem 3-4**

An experimental physicist submits a proposal to a granting agency requesting support to construct an infinite potential well analogous to Figure 3.5. Specifically, she proposes to build a well with L = 1 mm, inject some electrons into it and measure the wavelengths of photons emitted during "low-n" transitions via a spectrograph. As an expert on quantum mechanics, you are asked to evaluate the proposal. What is your recommendation?

From the Planck relation the wavelength of the photon $\lambda_{nj}$ emitted when a particle of mass m in an infinite well transits from state n to state j is given by

$$E_n - E_j = \frac{h^2}{8mL^2}(n^2 - j^2) = h\nu_{nj} = \frac{hc}{\lambda_{nj}},$$

or

$$\lambda_{nj} = \left(\frac{8mL^2c}{h}\right)\frac{1}{(n^2 - j^2)}.$$

For an electron in a well with $L = 10^{-3}$ m, this reduces to

$$\lambda_{nj} = \frac{3.297 \times 10^6}{(n^2 - j^2)} \text{ meters}.$$

Low n transitions will not yield photons in the visible range of wavelengths. A transition where $j = n-1$ that emits a photon of wavelength 5000 Å requires $n \approx 3.3 \times 10^{12}$, which means an energy of about $4.1 \times 10^{12}$ eV (4 million MeV). Reject the proposal.

3. Reed, Prob. 3-5

A particle in some potential is described by the wavefunction $\psi(x) = Axe^{-kx}$ $(0 \le x \le \infty;\ k > 0$; see Problem 2-5.) If $k = 0.5\text{Å}^{-1}$, what is the probability of finding the particle between x = 2.0 and 2.1Å?

From Problem 2.5 the normalization of this wavefunction is given by $A^2 = 4k^3$. Hence

$$P(x, x+\Delta x) = \psi^2(x)\Delta x = \left(A^2x^2e^{-2kx}\right)\Delta x = \left(4k^3x^2e^{-2kx}\right)\Delta x$$
$$= \left\{4(0.5\text{Å}^{-1})^3(2\text{Å})^2\exp\left[-2(0.5\text{Å}^{-1})(2.0\text{Å})\right]\right\}(0.1\text{Å}) = 0.0271.$$

4. Reed, Prob. 3-9

**Problem 3-9**

A proton is moving within a nuclear potential of depth 25 MeV and full width $2L = 10^{-14}$ meters. If the potential can be modeled as a finite rectangular well, how many energy states are available to the proton?

The number of states is given by

$$N(K) = 1 + \left[\frac{2K}{\pi}\right] = 1 + \left[\frac{2}{\pi}\sqrt{\frac{2mV_o L^2}{\hbar^2}}\right],$$

where the square brackets designate the largest integer less than or equal to the argument within. With $V_o = 25$ MeV $= 4.00 \times 10^{-12}$ J, $L = 0.5 \times 10^{-14}$ m, and a proton ($1.67 \times 10^{-27}$ kg),

$$N(K) = 1 + \left[\frac{2}{\pi}\sqrt{\frac{2(1.67 \text{ x } \text{x}10^{-27})(4.00 \text{ x } 10^{-12})(0.5 \text{ x } 10^{-14})^2}{(1.055 \text{ x } 10^{-34})^2}}\right]$$

$$= 1 + [3.487] = 4.$$

Only four energy states are available to the proton in this situation.