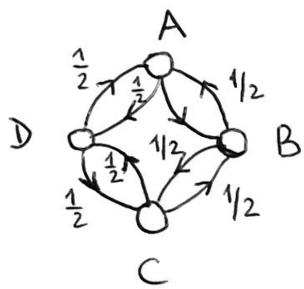
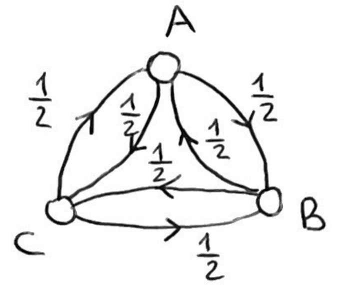
HW # 1

20 pt

(1.) Analyze $N_s = 3$ and $N_s = 4$ circular random walks defined by the following graphs:



$N_s = 4$



$N_s = 3$

The stochastic process is discrete-time and all transition probabilities are $\frac{1}{2}$.

(a) Use spectral methods to analyze both systems. Which system converges to the unique SS, and why? For the system with the SS, what are the SS probabilities?

(b) Use the transition prob. matrix $\mathbf{P}$ to explicitly simulate the system for $n = 10^3$ steps. Describe the "SS probabilities" found in this way in the light of your findings in part (a). Is there any dependence on the initial conditions?

(2.)

20 pt

Consider a two-state signal $X(t) = \{a, b\}$. The transitions between the two states are given by the following master equations:

$$\begin{cases} \partial_t P(a,t \mid x_0, t_0) = -\lambda P(a,t \mid x_0, t_0) + \\ \qquad + \mu P(b,t \mid x_0, t_0), \\ \partial_t P(b,t \mid x_0, t_0) = \lambda P(a,t \mid x_0, t_0) - \\ \qquad - \mu P(b,t \mid x_0, t_0). \end{cases}$$

Here, $P(a,t \mid x_0,t_0)$ is the prob. to be in state a at time t starting from state $x_0 \in \{a, b\}$ at time t_0:

$$P(x', t_0 \mid x_0, t_0) = \delta_{x' x_0}.$$

$P(b,t \mid x_0,t_0)$ is defined similarly.

This process is called the random telegraph process.

(a) Find $P(a,t \mid x_0,t_0)$ & $P(b,t \mid x_0,t_0)$.

Find the mean of $X(t)$:

$$m(t) = \sum_{x \in \{a,b\}} x \, P(x,t \mid x_0, t_0)$$

(b) Discuss the $t \to \infty$ limit of your findings in part (a). Are the results consistent with the S.S of the master equations?