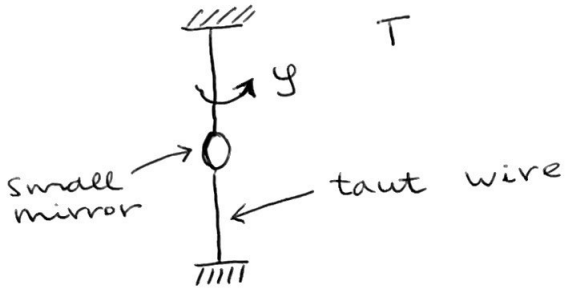
HW#3 (2025)

1. **10 pt** Consider a small mirror hanging on a thin vertical wire fixed at both ends and positioned in a room at $T = 287.1\ K$.



The angular Hooke's constant of the wire is $\kappa = 9.428 \times 10^{-9}\ \frac{g\,cm^2}{s^2}$. Using light beams, Kappler (in 1931) was able to measure the thermal average of φ^2, where φ is the angular deviation from equilibrium: $\langle \varphi^2 \rangle = 4.178 \times 10^{-6}\ rad^2$.

Use these data to compute the value of k_B, the Boltzmann constant.

What is the relative error wrt the modern value: $\epsilon = \frac{|k_B - k_B^{modern}|}{k_B^{modern}}$?

(2.) [10 pt] We know by direct calculation that $\frac{u}{V} = \frac{\pi^2}{15} \frac{(k_B T)^4}{(\hbar c)^3}$.

Using $pV = \frac{u}{3}$ and applying thermodynamics only, show that

$\frac{u}{V} \sim T^4$ [estimating the const prefactor requires the full calculation]

Hint: argue that p (pressure) depends only on T in this system.

(3.) [20 pt] Consider quantum spinless particles with the dispersion relation: $\epsilon(\vec{k}) = c|\vec{k}|$. The system is contained in a cubic d-dim box: $V = L^d$.

(a) Write down the expression for the grand canonical partition function (no need to evaluate the integral)

(b) Write down the expression for $\langle N \rangle$, the average number of particles. (again, leave it as an integral)

(c) Using the expression for $\langle N \rangle$ obtained above, argue that Bose-Einstein condensation only takes place if $d \geq 2$. What happens if $d = 1$?