HW#2 (2025)

(1.) $\boxed{10\text{ pt}}$ Adiabatic atmosphere

given air's low thermal conductivity, the atmosphere may be approximated as adiabatically insulated ideal gas at equilibrium.

Use the ideal gas equation of state, the law of adiabatic processes (see HW#1) and Stevin's law for the change of pressure with height z:

$$\frac{\partial p}{\partial z} = -mg\underbrace{\frac{N}{V}}_{\rho = \text{air density}}, \text{ where}$$

$$m \approx \frac{4m_{N_2} + m_{O_2}}{5} \text{ is air's average molecular mass}$$

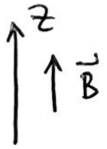
to prove that

$$\left(\frac{\partial T}{\partial z}\right)_S = -\frac{mg}{k_B}\left(1 - \gamma^{-1}\right), \text{ where } \gamma = \frac{c_p}{c_v}.$$

Hint: think of a column of air with unit cross-section and height z.

2. $\boxed{15\text{ pt}}$ Consider a system of N classical magnetic dipoles of magnitude μ and direction given by the spherical angles (θ, φ).
The dipoles have a moment of inertia I for rotations normal to their axis.

(a) Write the Lagrangian for this system when it is subjected to an external magnetic field $\vec{B} \uparrow\uparrow \hat{z}$:



(b) Write the corresponding Hamiltonian and evaluate the canonical partition function.

(c) Compute $m(B)$, the average magnetization per spin, at $T = \text{const}$.

3. $\boxed{15\text{ pt}}$ Consider a simple fluid described by a grand canonical ensemble. Express the following quantities in terms of the grand canonical partition function Σ:

$$\begin{cases} \langle \Delta E^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2, \\ \langle \Delta N^2 \rangle = \langle N^2 \rangle - \langle N \rangle^2, \\ \langle \Delta E \Delta N \rangle = \langle EN \rangle - \langle E \rangle \langle N \rangle. \end{cases}$$