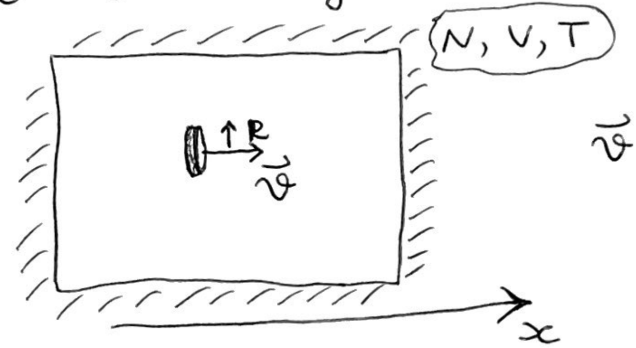
Final (2025)

1. Consider an ideal gas at temperature T, with particle density $\rho = N/V$ (N = number of particles, V = volume of the gas). Assume that the ideal gas consists of particles of mass m. Evaluate the drag force applied by the gas on a small disk of radius R moving at velocity v as shown:



$\vec{v} \uparrow\uparrow \hat{x}$

$R \ll \rho^{-1/3}$,

$v \ll$ average speed of particles in the gas

Hint: use Maxwell-Boltzmann distribution

2. Consider a system of non-interacting 2D fermions with spin $S = \frac{1}{2}$ and mass m, confined to a square box of area L^2.

(a) Evaluate the density of states $\rho(E)$.

(b) Evaluate the Fermi energy ϵ_F as a function of the particle density $\rho = \frac{N}{L^2}$.

(c) Evaluate the equation of state $P(\rho)$ as $T \to 0$.
pressure

(d) Find the chemical potential μ in the $T \to 0$ limit, to the leading order.

(3) Consider a system with free energy density

$$f(m) = d + \frac{1}{2} a m^2 + \frac{1}{4} b m^4 + \frac{1}{6} c m^6 - hm,$$

where $c > 0$ is fixed; m is magnetization

(a) Using $b > 0$ (fixed) and $h = 0$, find the extrema of $f(m)$ as a function of a, and discuss their stability. Discuss the (resulting) phases, find the location of the critical point in terms of a, and compute the order parameters.

(b) Using $b < 0$ (fixed) and $h = 0$, identify the minima of $f(m)$ and derive the order parameter of the broken-symmetry phase and the value of a at which the two phases co-exist.

(c) The point $(a, b, h) = (0,0,0)$ is called the tricritical point. Evaluate the critical exponent δ (defined by $h \sim |m|^\delta \operatorname{sgn}(m)$) in the vicinity of the tricritical point.

<u>Hint</u>: approach the tricritical point along the $a = b = 0$ hypersurface.