HW #4

① Training a neural network. (NN)

generate $N = 100$ equidistant data points using:

(a) $f(x) = x^2 + \xi, \quad x \in [-5, 5]$

(b) $f(x) = \sin(x) + \xi, \quad x \in [0, 2\pi]$

Here, $\xi = \mathcal{N}(\underbrace{0}_{\mu}, \underbrace{0.05^2}_{\sigma^2})$ is random noise.

Implement a two-layer feed-forward NN with M=5 hidden units. Choose the appropriate input & output units and justify this choice, with the goal of fitting $f(x)$ by regression.

Sketch (or plot) the NN architecture and train the NN by error backpropagation coupled with stochastic gradient descent.

Plot the data and the fit for both (a) & (b).

(2.) Bishop 5.26

(3.) Implement a binary Hopfield network (HN) with $I = 60$ spins. Choose 2 distinct memories $\vec{x}_1$ & $\vec{x}_2$ & compute w_{ij} by the Hebbian rule:

$$w_{ij} = \sum_{n=1}^{2} x_i^{(n)} x_j^{(n)} .$$

Each spin can adopt ± 1 values.

(a) Start $N=100$ runs from randomly chosen spin configurations and find the local energy minima of the energy function:

$$E = -\frac{1}{2} \sum_{i,j} w_{ij} x_i x_j \quad \text{[no biases]}$$

by asynchronous updates (cycle through all spins one at a time).

What are the # of times memories 1 and 2 were recovered?

(b) Now, set a quarter of all weights w_{ij} to $\emptyset$ randomly.

Repeat (a) starting from the <u>same</u> initial spin configurations & compare the # of times memories 1 and 2 were recovered with the results of (a).