HW #3 solutions

5.2

quantity, price

$D \geq Q$: profit $(P-C)Q$

demand, loss $\emptyset$, cost

$D<Q$: profit $(P-C)D$

loss $C(Q-D)$

Thus, expected profit + loss are:

$$E(Q) = \underbrace{\int_Q^\infty dD\, f(D)\,(P-C)\,Q}_{D \geq Q} +$$

$$+ \underbrace{\int_0^Q dD\, f(D)(P-C)D - \int_0^Q dD\, f(D)\,C(Q-D)}_{D<Q} =$$

D is a random var. with pdf $f(D)$ and cdf $F(D)$.

$$= (P-C)Q(1-F(Q)) + P\int_0^Q dD\, f(D)\, D - CQ\,F(Q).$$

Then $\frac{d}{dQ}E(Q) = (P-C)(1-F(Q)) - \cancel{(P-C)Qf(Q)} +$

$+\cancel{PQf(Q)} - CF(Q) - \cancel{CQf(Q)} = (P-C) - PF(Q) = 0,$ such that $Q \to Q^*$

$$F(Q^*) = \frac{P-C}{P},$$ as desired

6.2 Consider $p(x) \simeq q_\theta(x)$:

$$p(x) = q_\theta(x) + \underbrace{\Delta(x)}_{\ll 1,\ \forall x}.$$

Then

$$\underset{\substack{\uparrow \\ \text{KL divergence}}}{D}(p \| q_\theta) = \sum_x p(x) \log \frac{p(x)}{q_\theta(x)} \overset{\textcircled{=}}{}$$

$$\textcircled{=} \sum_x (\underset{\substack{\uparrow \\ \text{omit args for brevity}}}{q_\theta} + \Delta) \log\Big(1 + \overbrace{\frac{\Delta}{q_\theta}}^{\ll 1}\Big) \simeq \sum_x (q_\theta + \Delta)\Big[\frac{\Delta}{q_\theta} - \frac{\Delta^2}{2 q_\theta^2}\Big] \textcircled{\simeq}$$

$$\underset{\substack{\uparrow \\ \theta(\Delta^2)}}{\textcircled{\simeq}} \sum_x \Big[\Delta - \frac{\Delta^2}{2q_\theta} + \frac{\Delta^2}{q_\theta}\Big] \underset{\substack{\uparrow \\ \sum_x \Delta(x) = \sum_x (p(x) - q_\theta(x)) = 0}}{=} \sum_x \frac{\Delta^2}{2q_\theta}.$$

Thus, $D(p \| q_\theta) \simeq \frac{1}{2} \underbrace{\sum_x \frac{(p(x) - q_\theta(x))^2}{q_\theta(x)}}_{\text{"}\chi^2}$ in this case.