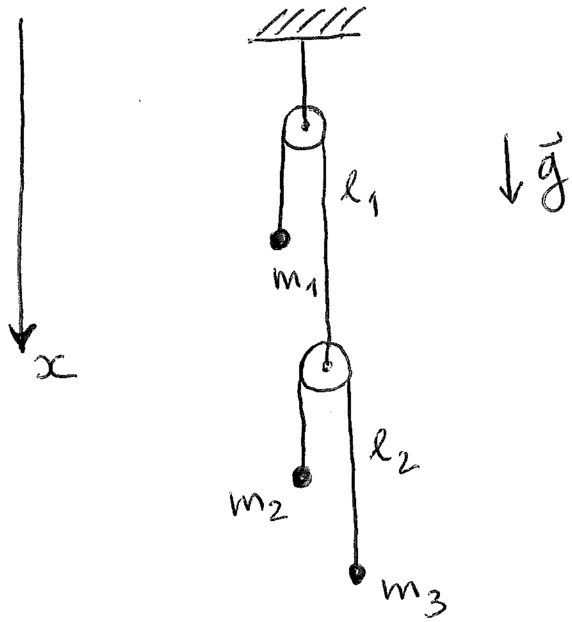
Final (2022)

1. [20 pt]

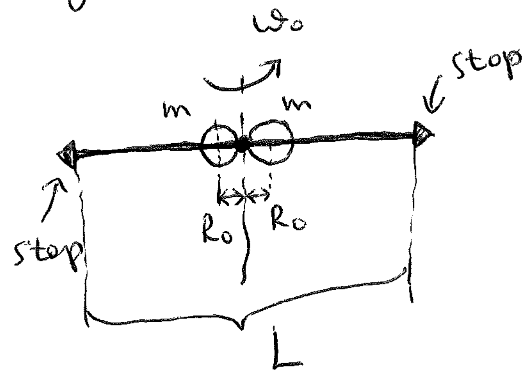
Consider a system of 3 weights suspended by pulleys:

l_1, l_2 = total rope lengths



Assume that the pulleys and the cords are massless and that there is no friction. Each pulley has a radius r and the coordinates of masses m_1, m_2, m_3 are x_1, x_2, x_3 respectively. Write down the equations of motion for all 3 masses. Solve the resulting equations with $x_i(t=0) = x_{i,0}$ and $\dot{x}_{i,0}(t=0) = 0$, $i=1,2,3$.

(2.) [20 pt] Consider a rotating horizontal rod of the total length L. Two spheres of mass m can glide along the rod as shown in the Fig. below:

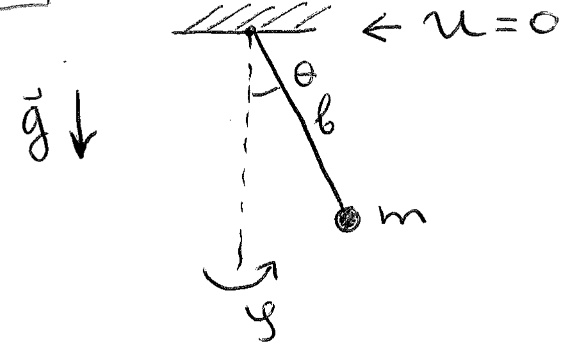


both spheres have mass m & radius R_0; each sphere has a centered hole through which the rod is passed. The rod's moment of inertia is I_0.

Imagine that the rod is given an initial angular velocity ω_0 and then the spheres are released from their position closest to the axis of rotation, shown in the Fig. above. assume that the spheres glide without friction and the collision with the stops at the ends of the rod is completely inelastic. Compute the final angular velocity ω_1 & the change in the rotational kinetic energy ΔT_{rot}. If $\Delta T_{rot} \neq 0$, where did the extra energy go or come from?

(3.) Consider a spherical pendulum of mass m and length b:

20 pt



Find the generalized momenta and write down (but do not solve!) Hamilton's EoM for this system. Comment on momentum conservation and find a cyclic coordinate (if any).

(4.) [20 pt]

Consider an arbitrary function $g(q, p, t)$, where $q = \{q_i\}_{i=1}^{n}$ are generalized coordinates and $p = \{p_i\}_{i=1}^{n}$ are canonical momenta.

(a) Write down an EoM for g using Poisson brackets

(b) Write down EoMs for p_j & q_j, $j = 1, \ldots, n$ using Poisson brackets

(c) Find $[p_i, p_j]$, $[q_i, q_j]$ and $[p_i, q_j]$

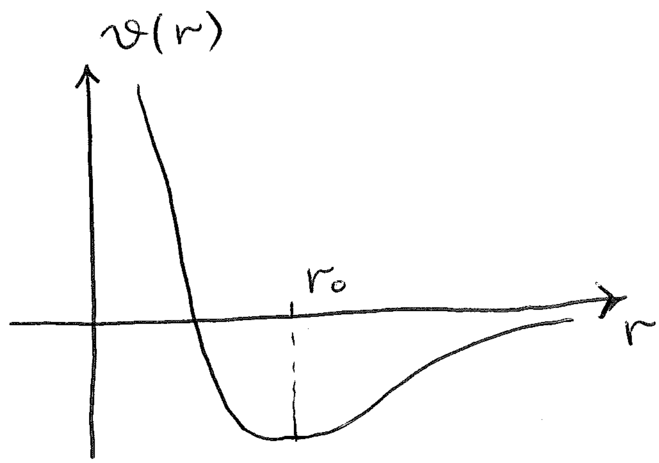
(d) Using results from (a), write down the conditions for g to be a constant of the motion of the system.

(5.) [20 pt]

The potential energy between two neutral atoms is given by:

$$v(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right],$$

where r is the separation distance.



Atom masses: m_1, m_2

(a) Find the equilibrium separation r_0.

(b) Find the frequency of small oscillations about the equilibrium position. Qualitatively, how will the average separation $\langle r \rangle$ vary with the amplitude of the oscillation? [this question does <u>not</u> assume small oscillations]