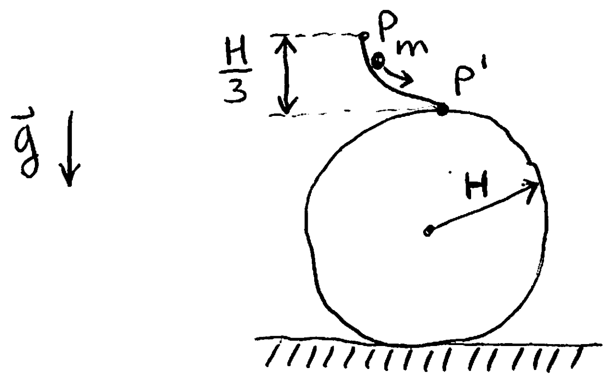
Final (2021)

(1.) Consider a sloping ramp and a sphere, as shown below:

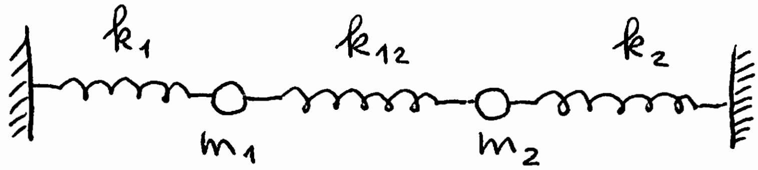


a particle of mass m starts from rest at the top of the ramp (point P) and slides down under the force of gravity, without friction or air resistance.

(a) what is the speed at point P'? Does this speed depend on the shape of the ramp? what about transit time from P to P' (explain qualitatively)

(b) At what height (above ground) does the mass lose contact with the slide?

(2.) Two masses are connected by three springs as shown:



k_1, k_{12}, k_2 are spring constants.

Let $x_1(t)$ and $x_2(t)$ be the displacements from equilibrium of m_1 and m_2, respectively.

(a) Write down the EoM for $x_1(t)$ and $x_2(t)$ (ignore gravity)

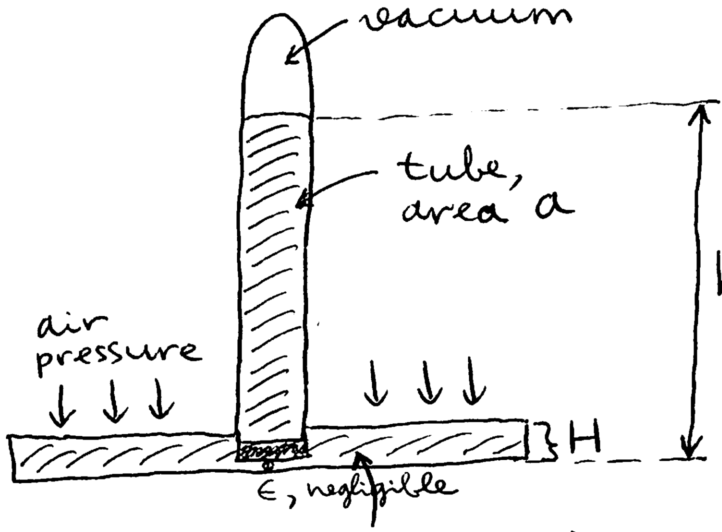
(b) Write down equations for the normal modes. Set $k_1 = k_{12} = k_2 = 2\ \frac{N}{m}$, $m_1 = m_2 = 0.5$ kg, and solve the normal mode equations, obtaining both the normal mode frequencies and relations between amplitudes.

(c) Using initial conditions

$$\begin{cases} x_1(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0, \\ x_2(0) = 0.1\ \text{m}, \end{cases}$$

write expressions for $x_1(t)$ and $x_2(t)$ using <u>real</u> numbers.

(3.) Consider a simple barometer: a fluid in a vertical tube with one end closed and the other end open to a reservoir, as shown:



ρ = density of the liquid

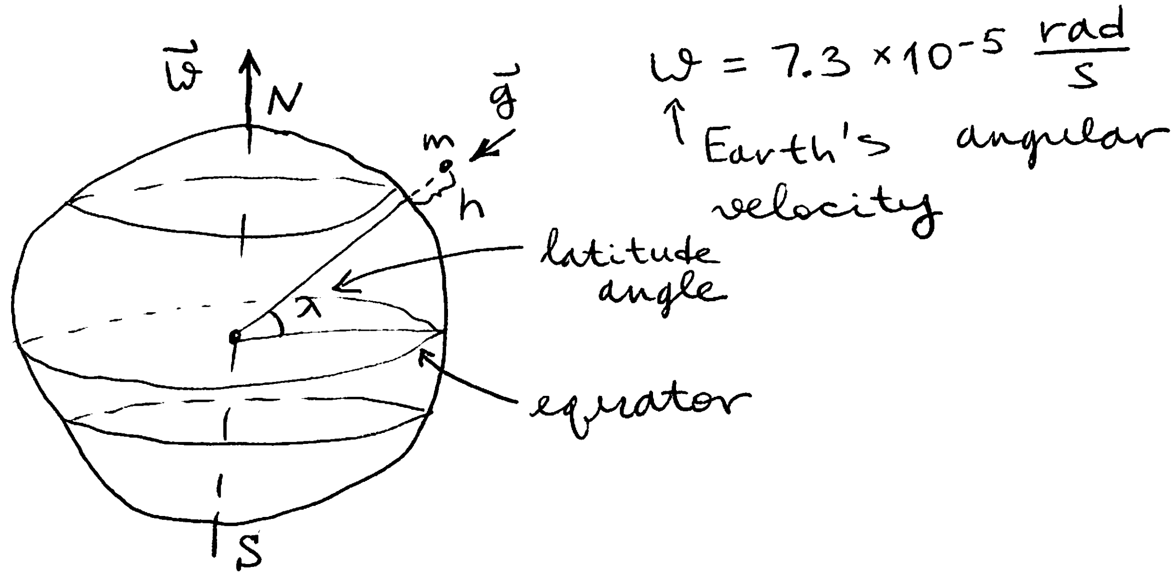
area ≡ cross-section

(a) Write down the Lagrangian of the system in terms of ρ, h, H, a. Derive the EoM from the Lagrangian. (assume that the liquid is frictionless)

(b) Compare the EoM with an equation for a simple harmonic oscillator. (SHO)

physically speaking, what are the main differences between this barometer and SHO? (discuss qualitatively)

(4.) Consider a particle of mass m falling freely in Earth's gravitational field from a height h. The particle (the particle is initially at rest) is situated in the Northern hemisphere, at the latitude λ:



(a) Find the horizontal deflection d of the particle due to the Coriolis force, to $\underline{\underline{O(\omega)}}$. Neglect air resistance.

(b) What is the deflection for a particle dropped from $h = 100$ m at $\lambda = 45°$?

(5.) Consider a scalar function

$$u = u(r^2, p^2, \vec{r}\cdot\vec{p}),$$ where

$\vec{r}$ is the radius-vector and $\vec{p}$ is the momentum of a single particle.

Show that $[u, L_z] = 0$ in this case,

↗ Poisson brackets

where L_z is the z-component of the angular momentum.

$$\begin{cases} \vec{r} = (x, y, z) \\ \vec{p} = (p_x, p_y, p_z) \end{cases}$$