

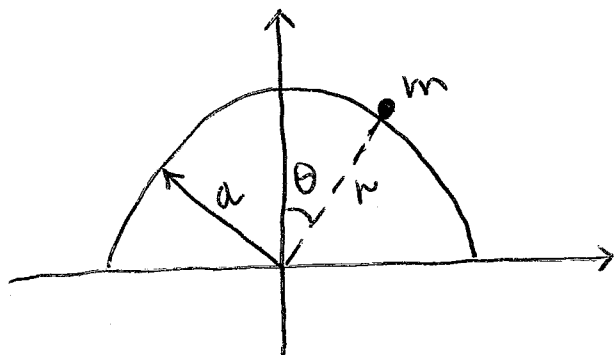
# Midterm (2021)

20 points

① a particle of mass  $m$  sits on top of a fixed frictionless hemisphere of radius  $a$ . The particle starts sliding down the hemisphere.

10 (a) Find the forces of constraint

10 (b) Find the angle  $\theta_0$  at which the particle leaves the hemisphere (i.e, the force of constraint becomes 0)



Hint: Use  $\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta} \dot{\theta}$  to solve the EoM

20 points

2. A satellite is in a circular orbit around Earth with radius  $R$ , velocity  $v$  and angular momentum  $l$ . The satellite accidentally fires its engine, giving it an outward radial velocity of the same magnitude  $v$  as the satellite's original velocity.

(a) Find the new total energy  $E_f$  and the new angular momentum  $l_f$ .

10 What type of orbit will the satellite be on?

10 (b) Discuss the subsequent motion of the satellite by plotting  $V_{\text{eff}}(r)$  and its components.

Note: assume that the satellite has an advanced engine design, s.t. firing the engine leads to a negligible loss of the satellite's mass  $m$ .