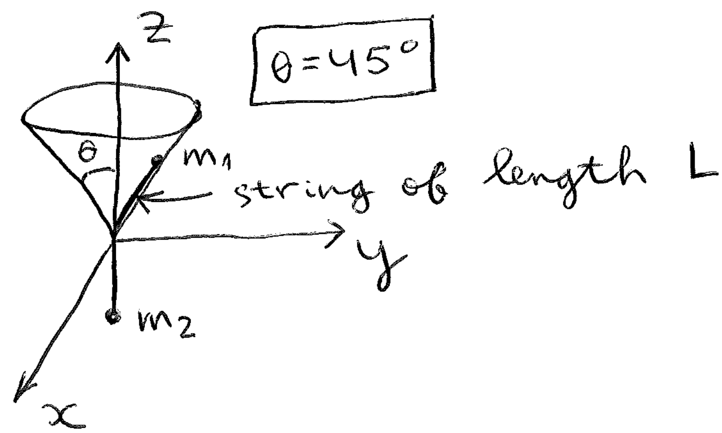
Midterm

1. Consider general 3D motion of a mass m_1 moving without friction on the inside of the cone surface (see Fig.). Another mass, m_2, is attached by a string of constant length L to m_1; m_2 can only move along the z axis. Write down the Lagrangian of the system, implementing all the constraints and carefully justifying the choice of generalized coordinates.



<u>Hint</u>: use cylindrical coordinates (r, φ, z) for m_1.

(2.) Consider a particle of mass m moving in a central force field defined by a potential $U(r)$. Assume that $U(r) \leq 0$ and the central force is pointing towards the origin.

(a) Show that the angular momentum of the particle $\vec{\ell}$ (defined wrt the origin of the force) is conserved.

(b) Assuming that $\vec{\ell} \neq 0$, argue that the particle's motion occurs in the plane which is $\perp \vec{\ell}$.

(c) Show that the total energy of the particle is given by

$$E = \frac{1}{2} m \dot{r}^2 + U_{eff}(r),$$

and find $U_{eff}(r)$.

(d) Find the condition for the circular orbit (if it exists), and write down an implicit equation for its radius r_0.

(e) Find the condition for the circular orbit to be stable.

(f) Consider $\vec{F} = -\left(\frac{b}{r^2} - \frac{c}{r^4}\right)\hat{r}$, where $b>0$, $c>0$ and $\hat{r}$ is the radial unit vector. Find the range of r_0 for which <u>stable</u> <u>circular</u> orbits are possible.

What happens to this range as $c \to 0$? Why? (i.e., explain the difference between $\vec{F}$ above and $\vec{F}' = -\frac{b}{r^2}\hat{r}$).