Physics 406
2012

HW # 6 Solutions

1. Θ Ch. 6, Q. 1

$\bar{e}$'s participating in the tetrahedral bond are in the valence band, from the point of view of the band structure. The "band" view and the "bond" view do not contradict each other because:

a) Bloch function can be both localized and periodic. ~~[illegible]~~

b) $\bar{e}$'s are not "assigned" to a particular bond in a crystal, but can contribute to any bond, in agreement with the Bloch function periodicity

2. Θ Ch. 6, Q. 4

Breaking of a bond corresponds to an $\bar{e}$ leaving the valence band and entering the conductance band; there is now a hole in the valence band.

3. θ Ch. 6, Q. 6

In (6.8),

$$n = \underbrace{2\left(\frac{m_e k_B T}{2\pi \hbar^2}\right)^{3/2} e^{E_F/k_B T}}_{\text{the prefactor is often called } g_{eff}\text{, the eff. DoS in CB}} e^{-E_g/k_B T}$$

This is justified since typically particle density is given by the DoS times the Boltzmann factor (here $e^{-E_g/k_B T}$)

4. θ Ch. 6, Q. 9

Intrinsic behavior is observed when $n_i \gg N_d - N_a$

In principle this inequality is satisfied if $N_d \simeq N_a$, so the sample is not necessarily pure.

5. $G = 0, \frac{2\pi}{a}$:

$$\psi_k(x) = C_k e^{ikx} + C_{k-\frac{2\pi}{a}} e^{i(k-\frac{2\pi}{a})x}$$

Using $q = k - \frac{G}{2}$ near BZB
(s.t. q is small),

we obtained

$$E^{\pm}(q) = E^{\pm}(0) + \frac{\hbar^2 q^2}{2m^*_{\pm}}, \text{ where}$$

$$m^*_{\pm} = m_0 \left\{ 1 \pm \frac{2}{U} \frac{\hbar^2}{2m_0} \left(\frac{\pi}{a}\right)^2 \right\}^{-1}, \text{ where}$$

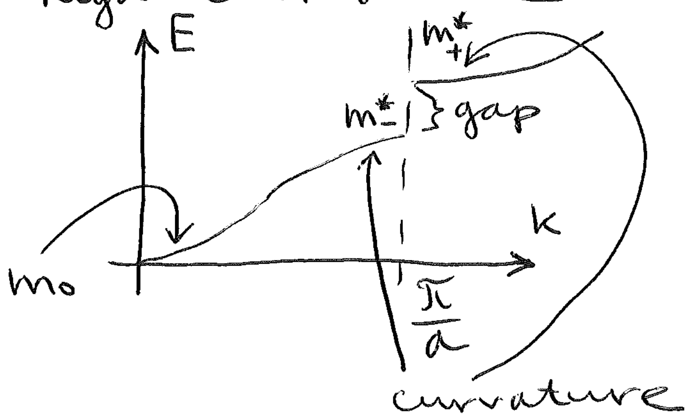
$U = U_G = U_{-G}$ is the Fourier component of the potential $(G = 0, \frac{2\pi}{a})$ and m_0 is the free e^- mass.

Presence of potential makes
$m^*_{\pm} \neq m_0$.

Note that $E^{\pm}(q) - E^{\pm}(0) = \underbrace{[\quad]}_{\text{curvature}} q^2$,

so curvature $\sim \frac{1}{m^*_{\pm}}$

high curvature $\rightarrow$ low eff. mass



If $k \simeq 0$, there is no Bragg-like reflection and so only $G=0$ is relevant:

$$(\lambda_k - E)\, C_k + \mathcal{U}\, C_k = 0, \text{ or}$$

$$E - \mathcal{U} = \frac{\hbar^2 k^2}{2m_0}.$$

Thus the mass is free $\bar{e}$ mass m_0 in the center of 1st BZ (under this model).

6. Omar Ch. 6, Pr. 2

Intrinsic sample of Si @ $T = 300\ K$:

$$\begin{cases} m_e = 0.7\, m_o \\ m_h = m_o \end{cases}$$

$$E_g \simeq 1.1\ eV$$

a) $$n = p = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_e m_h)^{3/4}\, e^{-E_g/2k_BT}$$

$$\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} = \left(\frac{1.4\times10^{-23}\times 300}{2\pi\ (1.05\times10^{-34})^2}\right)^{3/2} \simeq$$

$$\simeq 1.5\times10^{70}.$$

$$(m_e m_h)^{3/4} = (0.7\times(9.1\times10^{-31})^2)^{3/4} \simeq 6.6\times10^{-46}.$$

$$\frac{E_g}{2k_BT} = \frac{1.1\times1.6\times10^{-19}}{2(1.4\times10^{-23})\times300} \simeq 21.$$

$$e^{-E_g/2k_BT} \simeq 7.95\times10^{-10}.$$

So, $$n = p = 2\times1.5\times10^{70}\times(6.6\times10^{-46})\times$$
$$\times(7.95\times10^{-10}) \simeq 1.57\times10^{16}\,\frac{1}{m^3}.$$

b)

$$E_F = \frac{E_g}{2} + \frac{3}{4} k_B T \log \frac{m_h}{m_e} =$$

$$= \frac{1}{2}(1.1\ \mathrm{eV}) + \frac{3}{4} \underbrace{k_B T}_{0.026\ \mathrm{eV}} \log\left(\frac{1}{0.7}\right) \simeq$$

$$\simeq 0.557\ \mathrm{eV}.$$

7. <u>Omar Ch. 6, Pr. 5</u>

Si sample $N_d = 1 \times 10^{23} \frac{1}{m^3}$

$T = 300\ K$

a) In O Ch. 6, Pr. 2

we determined that

$n_{intrinsic} \simeq 10^{16} \frac{1}{m^3}$

So, $N_d >> n_{intrinsic}$

b) all impurities ionized:

$$n = N_d = 2 \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} e^{E_F/k_B T} e^{-E_g/k_B T}$$

$$\Downarrow$$

$$E_F = E_g + k_B T \log \left[\left(\frac{N_d}{2} \right) \left(\frac{2\pi \hbar^2}{m_e k_B T} \right)^{3/2} \right].$$

Using $E_g = 1.1\ eV$,

$k_B T \Big|_{T=300 k} = 0.026\, eV$

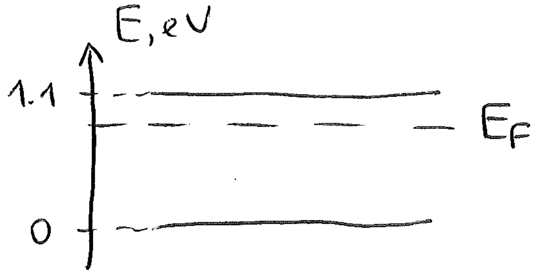
$$\left(\frac{2\pi \hbar^2}{\underset{0.7 m_0}{\underset{\shortparallel}{m_e}} k_B T} \right)^{3/2} = \left(\frac{2\pi \times (1.05 \times 10^{-34})^2}{0.7 \times (9.1 \times 10^{-31}) \times (1.4 \times 10^{-23}) \times 300} \right)^{3/2} \simeq$$

$$\simeq 1.32 \times 10^{-25}.$$

$$\log [\ldots] = \log \left[\frac{10^{23}}{2} 1.32 \times 10^{-25} \right] \simeq -5.02$$

Finally,

$$E_F = 1.1 - 5.02 \times 0.026 \text{ eV} \simeq 0.969 \text{ eV}$$



c) $N_a = 6 \times 10^{21} \frac{1}{m^3}$

$\underset{1 \times 10^{23} \frac{1}{m^3}}{\overset{\shortparallel}{N_d}} \gg N_a \Rightarrow$ the Fermi level will be shifted only slightly by acceptor impurities

Specifically,

$$n = \tilde{N}_d = N_d - N_a$$

$$E_F = E_g + k_B T \underbrace{\log\left[\left(\frac{\tilde{N}_d}{2}\right)\left(\frac{2\pi\hbar^2}{m_e k_B T}\right)^{3/2}\right]}_{<0}$$

$$\tilde{N}_d < N_d$$

$$\Downarrow$$

E_F will be slightly higher than with ($N_a = 0$, $N_d = 10^{23}\ m^{-3}$)

8. Omar Ch. 6, Pr. 11

a) Use $\frac{dN}{dE} = \frac{dN}{dV}\frac{dV}{dE}$

✓ Spherical energy surface:

$$V = \frac{4\pi}{3}k^3 = \frac{4\pi}{3}\left(\frac{2mE}{\hbar^2}\right)^{3/2}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\frac{dN}{dE} = \underbrace{\frac{1}{(2\pi)^3}}_{\frac{dN}{dV}} \underbrace{2\pi\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}}_{\frac{dV}{dE}} =$$

$$= \frac{1}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}.$$

✓ Ellipsoid:

$$V = \frac{4\pi}{3} k_\perp^2 k_z, \quad \text{where}$$

$$\begin{cases} k_\perp = \sqrt{\frac{2E}{\hbar^2} m_\perp}, \\ k_z = \sqrt{\frac{2E}{\hbar^2} m_z} \end{cases} \qquad \begin{cases} m_z \equiv m_e \\ m_\perp \equiv m_t \end{cases}$$

Then

$$\frac{dN}{dE} = \frac{1}{(2\pi)^3}\frac{d}{dE}\left(\frac{4\pi}{3}\frac{2m_\perp}{\hbar^2}\left(\frac{2m_z}{\hbar^2}\right)^{1/2}E^{3/2}\right) =$$

$$= \frac{1}{(2\pi)^2}\left(m_\perp^2 m_z\right)^{1/2}\left(\frac{2}{\hbar^2}\right)^{3/2}E^{1/2},$$ and

$$g(E) = \underset{\text{spin}}{2}\,\frac{dN}{dE}.$$

b) In Ge, $\begin{cases} m_\ell = 1.6\, m_0, \\ m_t = 0.08 m_0 \end{cases}$

Then $m_d = \left(m_t^2 m_\ell\right)^{1/3} =$

$$= \left((0.08)^2 \times 1.6\right)^{1/3} m_0 \simeq 0.22\, m_0.$$

9. Recent developments in the area of nanowires/nanotubes/nanoelectronics:

1) Single-electron transistors composed of metallic carbon nanotubes

2) Self-assembly of nanowires & nanoarrays

3) Use as ultrasensitive detectors to detect gas molecules & biochemical compounds

4) Molecular electronics