Physics 406
2013

HW #8
Solutions

① Omar Q. 2

Eq. (10.12):

$$H_c(0) \simeq \left(\frac{2 n k_B^2}{\mu_0 E_F}\right)^{1/2} T_c$$

Recall that in 3D,

$$E_F \sim n^{2/3}$$

Thus $H_c(0) \sim n^{1/6} T_c$, the dependence on n is very weak

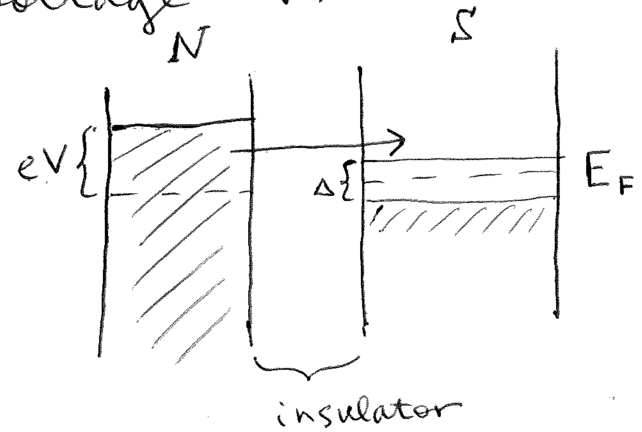
$H_c(0) \sim T_c$ for any superconductor

(2.) Θ Ch. 10, Q. 4.

For frequencies $\omega > \frac{\Delta}{\hbar}$, there will be some normal $\bar{e}$'s present which have finite scattering times. Then the resistivity will be finite, as in a normal metal.

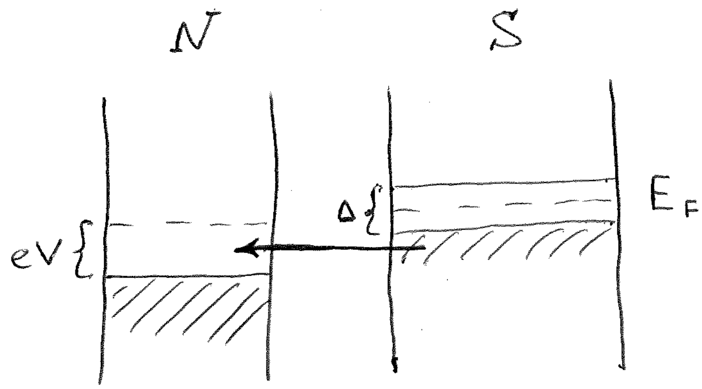
(3.) Θ Ch. 10, Q. 7

We will assume that in an electrically symmetric junction the magnitude of the current should be independent of the sign of external voltage V.



As is clear from the diagram, $\bar{e}$ tunneling will occur once $eV > \frac{\Delta}{2}$ (E_F is in the middle of Δ)

Application of reverse voltage will lead to a current also:



Its sign is reversed but the magnitude is independent of the sign of V, so the junction is electrically symmetric.

(4) θ Ch. 10, Pr. 1

Solenoid:

$$L \frac{dI}{dt} + IR = 0,$$

$$\frac{dI}{I} = -\underbrace{\frac{R}{L}}_{\equiv \Gamma} dt \Rightarrow I(t) = I(0)\, e^{-\Gamma t},$$

where Γ is the damping timescale

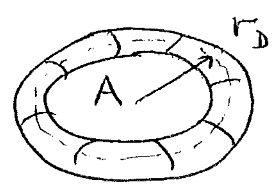
Current damped by 0.01%:

$$\frac{I}{I(0)} = 0.9999 = e^{-\Gamma \tau}, \text{ or}$$

$$\tau = -\frac{\log(0.9999)}{\Gamma}.$$

Now, $R = \rho \dfrac{l_w}{A} = \rho \dfrac{l_w}{\pi\left(\frac{d_w}{2}\right)^2}$.

$$L = \frac{\varphi}{I} = \frac{\mu_0 N^2 A}{2\pi r_D}$$



$\mu_0 N I = B(2\pi r_D)$, or

\# turns

$$B = \frac{\mu_0 N I}{2\pi r_D}.$$

Then $\varphi = NBA = \dfrac{\mu_0 N^2 A}{2\pi r_D} I$

So, $\underbrace{\dfrac{R}{L}}_{\Gamma} = \dfrac{\rho l_w}{\pi\left(\frac{d_w}{2}\right)^2} \times \dfrac{2\pi r_D}{\mu_0 N^2 A}$.

Not all #'s are given $\Rightarrow$ leave in symbolic form

(5.) θ Ch. 10, Pr. 9

Uncertainty principle:

$$\Delta x \, \Delta p \sim \hbar, \text{ or}$$

$$\Delta x \sim \frac{\hbar}{\Delta p}.$$

Only $\bar{e}$'s within $k_B T_c$ of E_F can participate in the development of ~~supe~~ superconductivity:

$$\Delta E \sim k_B T_c,$$

$$E = \frac{p^2}{2m} \rightarrow \Delta E = \frac{p \Delta p}{m} \simeq v_F \Delta p, \text{ or}$$

$$\Delta p \sim \frac{k_B T_c}{v_F}.$$

Then $$\xi_0 \sim \Delta x \sim \frac{\hbar v_F}{k_B T_c} \sim \frac{\hbar v_F}{\Delta}.$$

Typically $\xi_0 = \theta(10^2)$ nm,

e.g. Al $\xi_0 = 160$ nm,
Cd $\xi_0 = 76$ nm.

(6.) Key points on the intermetallic superconductor MgB_2.

a) Original idea: investigation of ternary compounds with 2 light atoms (Mg & B) and one intermediate one (Ti). Expected large DoS $\Rightarrow$ large T_c.

b) Discovered that $T_c \sim 40K$ is actually due to binary compound MgB_2.

c) MgB_2 wires can be made relatively easily $\Rightarrow$ practical applications.

d) Observation of the isotope effect in MgB_2 $\Rightarrow$ role of $\bar{e}$-phonon interactions.

e) Combination of high critical field & low normal-state resistivity make MgB_2 attractive for making sc magnets.

f) mechanism of sc unknown $\Rightarrow$ two superconducting gaps?