Physics 406

HW #6 solutions

1. θ. Ch. 5, Pr. 12 (1D)

a) $k = \frac{2\pi}{L} n \Rightarrow N = \frac{k}{(2\pi/L)} = \frac{Lk}{2\pi}$

N ↑ # states with $\leq k$

$$\frac{dN}{dk} = \frac{L}{2\pi} = \frac{1}{2\pi} \quad \text{ile } L = 1$$

$$g(E) = \frac{dN}{dE} = \frac{dN}{dk}\frac{dk}{dE} = \frac{1/2\pi}{(dE/dk)}$$

b) TB model: (5.43) in θ

$$E(k) = E_0 + 4\gamma \sin^2\left(\frac{ka}{2}\right)$$

Then

$$\frac{dE}{dk} = 8\gamma \sin\left(\frac{ka}{2}\right)\cos\left(\frac{ka}{2}\right) \times \frac{a}{2} =$$

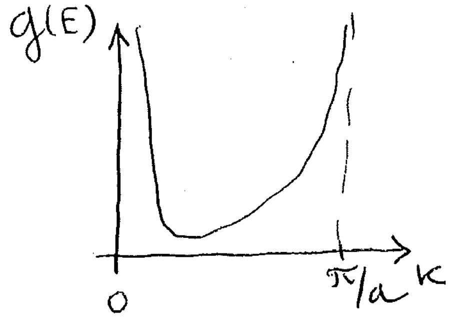
$$= 2\gamma a \sin(ka)$$

So, $$g(E) = \frac{1/2\pi}{2\gamma a \sin(ka)}$$

Limits:

$k \to 0$: $g(E) \sim \frac{1}{k} \sim \frac{1}{\sqrt{E(k) - E_0}}$

$k \to \frac{\pi}{a}$: $g(E) \to \infty$



2. ⊖ Ch. 6, Q. 1

$\bar{e}$'s participating in the tetrahedral bond are in the valence band, from the point of view of the band structure. The "band" view and the "bond" view do not contradict each other because:

a) Bloch function can be both localized and periodic.

b) $\bar{e}$'s are not "assigned" to a particular bond in a crystal, but can contribute to any bond, in agreement with the Bloch function periodicity

3. ⊖ Ch. 6, Q. 4

Breaking of a bond corresponds to an $\bar{e}$ leaving the valence band and entiring the conductance band; there is now a hole in the valence band.

4. Ch. 6, Q. 9

Intrinsic behavior is observed when $n_i \gg N_d - N_a$

In principle this inequality is satisfied if $N_d \simeq N_a$, so the sample is not necessarily pure.

5. O. Ch. 6, Q. 12

It might be possible to create $\bar{e}$ gas with T lower than that of the lattice. Hypothetically, one might use adiabatic expansion of $\bar{e}$ gas, or evaporative cooling using a quantum well potential barrier.

to allow "hotter" $\bar{e}$'s to escape

Any other ideas?

6. Omar Ch. 6, Pr. 2

Intrinsic sample of Si @ T = 300 K:

$$\begin{cases} m_e = 0.7\, m_0 \\ m_h = m_0 \end{cases}$$

$E_g \simeq 1.1$ eV

a) $n = p = 2 \left(\frac{k_B T}{2\pi\hbar^2} \right)^{3/2} (m_e m_h)^{3/4} e^{-E_g/2k_BT}$

$$\left(\frac{k_B T}{2\pi\hbar^2} \right)^{3/2} = \left(\frac{1.4\times10^{-23}\times300}{2\pi\,(1.05\times10^{-34})^2} \right)^{3/2} \simeq$$

$$\simeq 1.5\times10^{70}.$$

$$(m_e m_h)^{3/4} = \left(0.7\times(9.1\times10^{-31})^2\right)^{3/4} \simeq 6.6\times10^{-46}.$$

$$\frac{E_g}{2k_BT} = \frac{1.1\times1.6\times10^{-19}}{2(1.4\times10^{-23})\times300} \simeq 21.$$

$$e^{-E_g/2k_BT} \simeq 7.95\times10^{-10}.$$

So, $n = p = 2\times1.5\times10^{70}\times(6.6\times10^{-46})\times$
$\times(7.95\times10^{-10}) \simeq 1.57\times10^{16}\ \frac{1}{m^3}$.

b)

$$E_F = \frac{E_g}{2} + \frac{3}{4} k_B T \log \frac{m_h}{m_e} =$$

$$= \frac{1}{2}(1.1\,eV) + \frac{3}{4} \underbrace{k_B T}_{0.026\,eV} \log\left(\frac{1}{0.7}\right) \simeq$$

$$\simeq \underline{\underline{0.55\overset{7}{\cancel{8}}\ eV}}.$$

7. Omar Ch. 6, Pr. 5

Si sample $N_d = 1 \times 10^{23} \frac{1}{m^3}$

$T = 300\ K$

a) In O Ch. 6, Pr. 2
we determined that
$n_{intrinsic} \simeq 10^{16} \frac{1}{m^3}$

So, $N_d >> n_{intrinsic}$

b) all impurities ionized:
$$n = N_d = 2 \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} e^{E_F/k_BT} e^{-E_g/k_BT}$$
$$\Downarrow$$
$$E_F = E_g + k_BT \log \left[\left(\frac{N_d}{2} \right) \left(\frac{2\pi\hbar^2}{m_e k_B T} \right)^{3/2} \right].$$

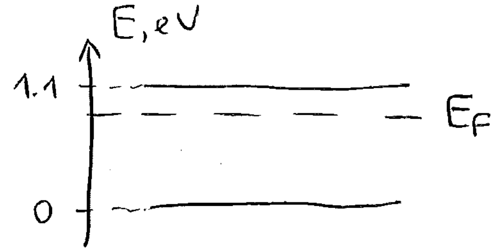
Using $E_g = 1.1\ eV$,
$k_BT \Big|_{T=300k} = 0.026\ eV$

$$\left(\frac{2\pi\hbar^2}{\underset{\underset{0.7 m_0}{\parallel}}{m_e} k_B T} \right)^{3/2} = \left(\frac{2\pi \times (1.05 \times 10^{-34})^2}{0.7 \times (9.1 \times 10^{-31}) \times (1.4 \times 10^{-23}) \times 300} \right)^{3/2} \simeq$$
$$\simeq 1.32 \times 10^{-25}.$$

$$\log [\ldots] = \log \left[\frac{10^{23}}{2}\ 1.32 \times 10^{-25} \right] \simeq -5.02$$

Finally,

$$E_F = 1.1 - 5.02 \times 0.026 \text{ eV} \simeq 0.969 \text{ eV}.$$



c) $N_a = 6 \times 10^{21} \frac{1}{m^3}$

$N_d \gg N_a \Rightarrow$ the Fermi level will be shifted only slightly by acceptor impurities

$\overset{\shortparallel}{1 \times 10^{23} \frac{1}{m^3}}$

Specifically,

$$n = \tilde{N}_d = N_d - N_a$$

$$E_F = E_g + k_B T \underbrace{\log\left[\left(\frac{\tilde{N}_d}{2}\right)\left(\frac{2\pi\hbar^2}{m_e k_B T}\right)^{3/2}\right]}_{<0}$$

$$\tilde{N}_d < N_d$$

$$\Downarrow$$

E_F will be slightly higher than with ($N_a = 0$, $N_d = 10^{23}\ m^{-3}$)

8. Q. Ch. 6, Pr. 6

Si: $\begin{cases} \mu_e = 1350 \ \frac{cm^2}{V \cdot s} \\ \mu_h = 475 \ \frac{cm^2}{V \cdot s} \end{cases}$ $E_g = 1.1 \, eV$

$m_h = m_0$,

$m_e = 0.7 \ m_0$

a) $\bar{e}/h$ lifetimes

$$\mu_e = \frac{e\tau_e}{m_e} \Rightarrow \tau_e = \frac{\mu_e m_e}{e} =$$

$$= \frac{(1350 \times 10^{-4}) \times (0.7) \times (9 \times 10^{-31})}{1.6 \times 10^{-19}} \simeq$$

$$\simeq 5.3 \times 10^{-13} \ s.$$

$$\tau_h = \frac{\mu_h m_h}{e} = \frac{\mu_h}{\mu_e} \frac{m_h}{m_e} \tau_e =$$

$$= \frac{475}{1350} \frac{1}{0.7} 5.3 \times 10^{-13} \ s \simeq 2.7 \times 10^{-13} s.$$

b) $$\sigma = ne(\mu_e + \mu_h) =$$

$$= \underbrace{(1.57 \times 10^{16})}_{\text{n from Pr. 2}} \times (1.6 \times 10^{-19}) \times (1350 + 475) \times 10^{-4} \simeq$$

$$\simeq 4.4 \times 10^{-4} \ (\text{Ohm} \times m)^{-1}$$

@ room T

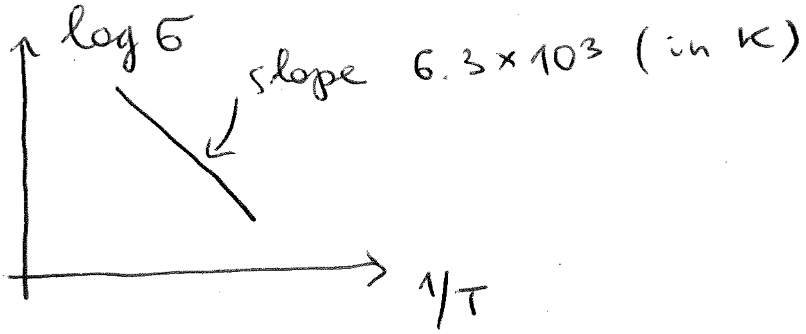
c) $\sigma(T)$ in intrinsic regime:

$$\sigma = f(T)\, e^{-E_g/2k_BT}, \text{ or}$$

$$\log \sigma = \log f(T) - \frac{E_g}{2k_BT}$$

For Si, $\dfrac{E_g}{2k_B} = \dfrac{1.1\ eV}{2\times(8.6\times10^{-5}\ eV/K)} =$

$$= 6.3\times10^{3}\ \frac{1}{T}$$



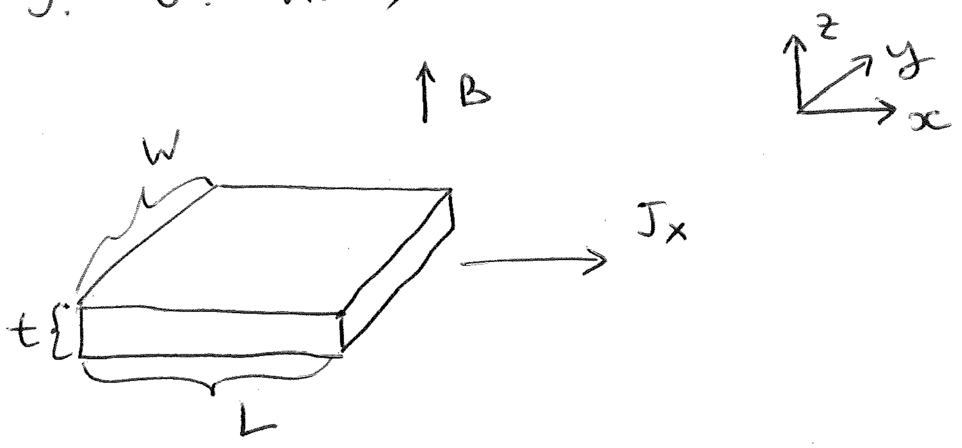
phonon scattering $\Rightarrow \mu \sim \dfrac{1}{T^{3/2}}$

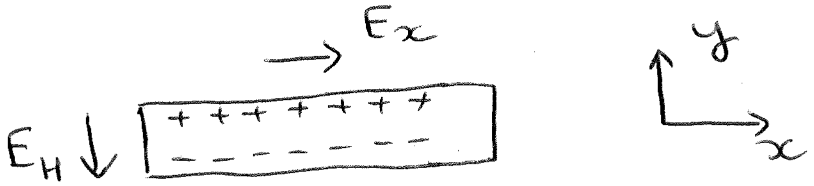
Prefactors in n (or p):

$$n = p \sim T^{3/2}.$$

So, $f(T) = \text{const}(T)$ & $\log \sigma$ is a straight line, as shown above.

9. Q. Ch. 6, Pr. 8





$L = 5\,cm$

$t = 1\,mm$ (thickness)

$W = 0.5\,cm$

$B = 0.6\ Wb/m^2$

$V_H = 8\,mV$

$J_x = 10\,mA$

$$R_H = \frac{E_y}{J_x B_z}$$

Assume a dominant carrier type (sign of R_H will tell us holes or $\bar{e}$'s):

$$E_y = \frac{V_H}{W} = \frac{8\,mV}{0.5\,cm} = 1.6\ \frac{V}{m}.$$

$$R_H = \frac{E_y}{J_x B_z} = \frac{1.6}{10^{-2} \times 0.6} = 2.6 \times 10^{2} \ \frac{V \cdot m^3}{amp \cdot Wb}$$

$R_H > 0 \Rightarrow$ p-type (holes dominate)

Then $R_H = \frac{1}{pe} \Rightarrow$

$$\Rightarrow p = \frac{1}{R_H e} = \frac{1}{(2.6 \times 10^{2})(1.6 \times 10^{-9})} \simeq$$

$$\simeq 2.4 \times 10^{16} \ \frac{1}{m^3}.$$

Mobility $\mu_H = \sigma R_H$,

$$\sigma = \frac{J_x}{E_x}$$

E_x (or σ) not given $\Rightarrow$ cannot compute μ_H.

10. Recent developments in the area of nanowires / nanotubes / nanoelectronics:

1) Single-electron transistors composed of metallic carbon nanotubes

2) Self-assembly of nanowires & nanoarrays

3) Use as ultrasensitive detectors to detect gas molecules & biochemical compounds

4) Molecular electronics