

Mach's Principle and an Inertial Solution to Dark Matter

by

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ABSTRACT

We introduce a novel way to implement Mach's Principle, by using the kinematics of inertial motion in a local reference frame, and derive a dynamical inertial force term in a straightforward manner. Using observational data from large scale structures in the visible Universe, we find that the fundamental equation of MOND is produced simply, and naturally. Moreover, with no free parameters, the MOND acceleration magnitude is determined uniquely. Thus, the puzzling existence of ubiquitous, flat galactic rotation curves can be explained in a truly physical (as opposed to phenomenological) fashion without using dark matter, relying solely on the observed distribution of large-scale structure in the Universe.

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I. KINEMATICS AND DYNAMICS USING MACH'S PRINCIPLE

Few, if any, ideas in physics have generated more confusion than Mach's Principle. His notorious distrust of theory, coupled with a penchant for philosophizing in sometimes ambiguous non-mathematical terms, has resulted in hundreds of interpretations of what is even meant by the term. *Zeitschrift für Physik* even refused papers on the subject, because of the ubiquity of rampant polemical replies (Pais, 1982). About the only premise agreed upon is that Mach explicitly states that the motion of any test body is to be defined with respect to the rest of the entire Universe. To implement this, we visualize an idealized 2-body system, generalizing shortly to n-bodies below.

Initially, we consider a 2-body system exhibiting "inertial" motion, as in Figure 1.

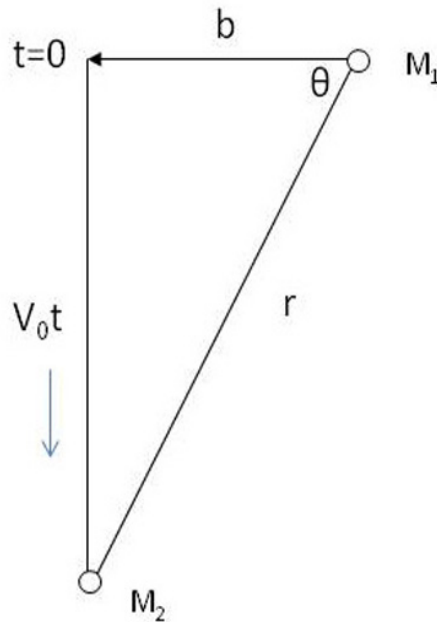


Figure 1: Idealized inertial motion in a two body system with constant V_0 .

We see that in polar coordinates using M_1 as origin:

$$r^2 = b^2 + (V_0 t)^2 \quad \text{and}$$

$$r \, dr = V_0^2 \, t \, dt, \quad \text{so that}$$

$$d^2r/dt^2 = 1/r [V_0^2 - (dr/dt)^2] \quad 1)$$

It is quite amusing to note how coyly Mach considered this inertial velocity V_0 . Of course, he couldn't call it that since it would be a Newtonian reference to absolute space. Instead, he calls it "a", describing it as a "constant dependent on the directions

and velocities..." (Mach, 1919) thus maintaining a relativist approach. Let us pursue this further.

A material point which is supposedly unaffected by external forces (such as gravity) moves in a straight line at constant speed only relative to a distinguished set of co-ordinate systems. Such a system cannot be assured locally, as we see above. The GR approach dictates that the way out of this conundrum is via geometry: a correct metric should accommodate any co-ordinate system, however bizarre or pathological. What is interesting in this case is that Newton's law of inertia breaks down precisely when it ought to be easiest! I.e. when $\Delta V = 0$. Why is that? Because as soon as you use your local environment to define your coordinates (and hence V_0) you introduce three inertial terms: centrifugal, transverse, and Coriolis (as can be seen easily by deriving the equation of motion for θ).

However, requirements of general covariance demand that we take Eq. 1 seriously, especially if the 2 bodies are local. (We can operationally define "local" as 2 bodies that admit of a non-zero measurement of "b" in Figure 1. If $b=0$, the objects are sufficiently far away so that motion is purely radial, $V_0 \rightarrow dr/dt$, and our co-ordinates become "inertial".) Newton avoids this "problem" altogether by merely taking M_1 and M_2 as completely unrelated to each other, and taking $V_0 = \text{constant}$ as being defined by an inertial system at essentially infinity. Mach, of course, says no to this approach. The problem becomes one of constructing a formalism in which motions of bodies are influenced by each other, but in which the concept of an inertial frame is not introduced *a priori*. But if there is a relationship between M_1 and M_2 , what is its dynamical nature?

We note that any two masses M_2 and M_2' with differing distances from M_1 but with identical V_0 ought to be able to preserve their direction and magnitude independent of placement in space (i.e. independent of $\vec{r}$). Thus, if we imagine these masses at some moment in line with $\vec{b}$ and $\vec{b}'$ and moving with the pictured velocity V_0 , (i.e. co-linear at $t, t'=0$), $dr/dt = dr'/dt' = 0$. Therefore, in order to preserve the constancy of V_0 in both cases, Equation 1 requires any putative dynamical force related to the masses to be proportional to $1/r$. Only if

$$|F| \propto 1/r \quad 2)$$

can we ensure that any possible inertial "force" be universal and capable of maintaining a supposedly constant V_0 , independent of position in space. (It is perhaps prudent at this point to point out that we do not have any direction defined yet, since Eq. 1 is a scalar relationship.) Interestingly, this $1/r$ dependence is the functional form suggested by Sciama (1953) and Ciufolini and Wheeler (1995), in their attempts to induce inertia via radiation fields. Unmodified gravity cannot fundamentally do this. Its r^{-2} dependence means that some function of r must remain in the kinematics. In the absence of a metrical change (which is probably inevitable for a relativistic implementation), only a dynamical force proportional to $1/r$ can save the phenomenon of a constant V_0 .

Furthermore, various elementary observational requirements demand from central forces that $|F| \propto M_i m_k$. Now, our task is to implement a vector position. The only

means at our disposal to extract direction is to appeal to the equivalence principle, which at least *allows* us to use "G" as a universal constant, as a linkage to gravity, via $m_{\text{inertial}} \equiv m_{\text{grav}}$. Thus, we can utilize $GM_j m_k$ as a valid factor to extend Eq. 2. Thus,

$$\vec{F}_m \propto GM_i m_k / \vec{r}_{ik} \quad 3)$$

So, if there is *any* hitherto unknown connection between M_{inertial} and M_{grav} , the inertial aspect must be related to gravity via *direction* toward distant objects, since inertia appears to be independent of local position, in the sense that V_0 is arbitrary in both magnitude and orientation with respect to any and all local mass elements present in the dynamical system. In short, we appeal to classical gravity's vector nature to provide direction for our putative inertial condition.

Clearly, in 2-body systems, the direction of gravitational acceleration is always unique, but when we proceed to n-body systems, complications arise. All we can say, in an arbitrary n-body system, is if we observe the local direction of gravitational acceleration (introducing a test particle at any point in spacetime), this direction will always contain, as time goes on, a component toward the source of the dominant gravitational field.

In order to allow a direction to be thus defined everywhere by gravity, it is therefore reasonable to choose the maximum gravitational/inertial extent of an object as the directional determining parameter necessary to bring Eq. 3 into a dimensionally correct form. This directional extent, which we designate as u_i , becomes the measure of inertial "influence" of any M_i . It determines the limiting distance beyond which the gravitational field of M_i is no longer the strongest in the system. Only if $r_{ik} < u_i$ will any test mass m_k have at least a component of its motion toward M_i . For $r_{ik} > u_i$, the mass m_k will move towards another direction determined by the position of M_j where now $r_{jk} < u_j$, and which thus determines the source (and direction) of the dominant gravitational/inertial mass acceleration. Therefore, the proposed force in Equation 3 becomes:

$$\vec{F}_m = - \frac{G M_i}{u_i} * \frac{m_k}{\vec{r}_{ik}} \quad (\text{if attractive}) \quad 4)$$

The existence of u_i for any system of M_i merely implies the fact that an equilibrium value of the center of mass between any two objects can be found. In a sense, u_i defines a Lagrange point for inertial interactions. But, there is absolutely no reason *a priori* to expect that these values will have any functional significance. For example, in a uniformly populated distribution of stars of equal mass, u_i will be invariant and equal to about half the interstellar spacing of the distribution. To take another example, we imagine the 3 body system pictured in Figure 2.

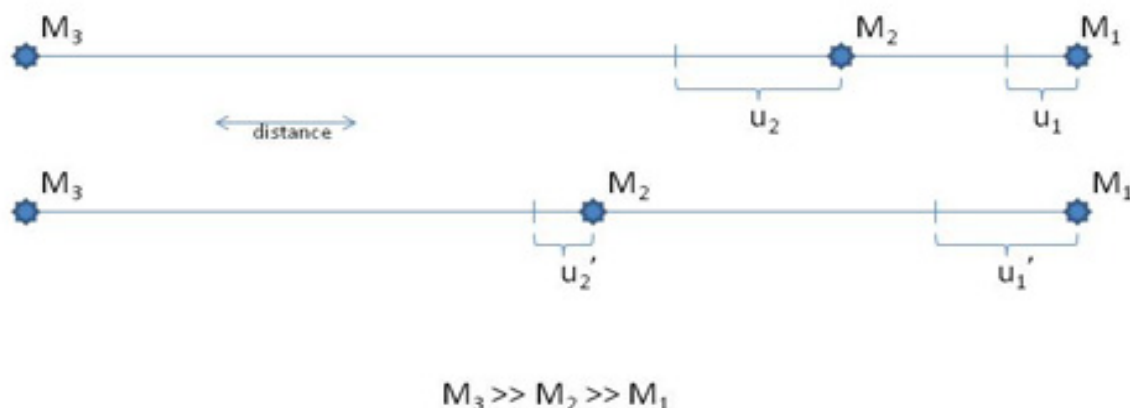


Figure 2. Showing changes in u_i for two different simple distributions of 3 masses.

This schematic diagram clearly indicates the fact that we can alter the magnitude of u_i significantly and apparently at will, simply by altering the relative positions of the M_i . Any systematic distribution of u_i and hence M_i in the Universe would be truly surprising.

II. LARGE SCALE STRUCTURE AND MOND

One of the more remarkable (yet seemingly trivial) property of the Universe is that it *doesn't* consist of a uniform sprinkling of individual masses. For example, long before we get to the half-way point from the Sun to, say, α Centauri (which we might naively assume to approximate u_{sun}), we encounter the dominant field of the Milky Way itself. Beyond the u value quoted below for the Milky Way, to take yet another example, the Virgo cluster becomes the dominant gravitational source rather than M31.

For extended systems, such as globular clusters, galaxies, etc., we can approximate u_i by the observed physical size of the object itself. In this fashion, u becomes a tidal radius. For systems like the Sun, of course, we need to resort to the maximum influence distance as elucidated above. The results are shown in Table 1 and Figure 3 below:

	<u>M</u>	<u>u</u>	<u>M/u</u>
Sun:	2×10^{33} g	8×10^{16} cm	2.5×10^{16} g/cm
Globular clusters:	2×10^{38} g	5×10^{19} cm	4×10^{18} g/cm
Dwarf Spheroidals:	2×10^{41} g	2×10^{21} cm	1×10^{20} g/cm
Milky Way:	2×10^{44} g	1×10^{23} cm	2×10^{21} g/cm
Coma Cluster:	6×10^{47} g	1×10^{25} cm	6×10^{22} g/cm

Notes: value for dSph is the tidal radius, GC's use $r=0.9$ total light, Coma mass is x-ray gas mass, with $r=3$ Mpc (isothermal sphere value), Milky Way shows radius of the globular cluster system ~ 30 kpc, Sun uses distance at which Sun's gravitational field equals that of the Milky Way.

Table 1: All values of mass and size are taken from Carroll and Ostlie (2006). Moreover, only *luminous* masses are quoted.

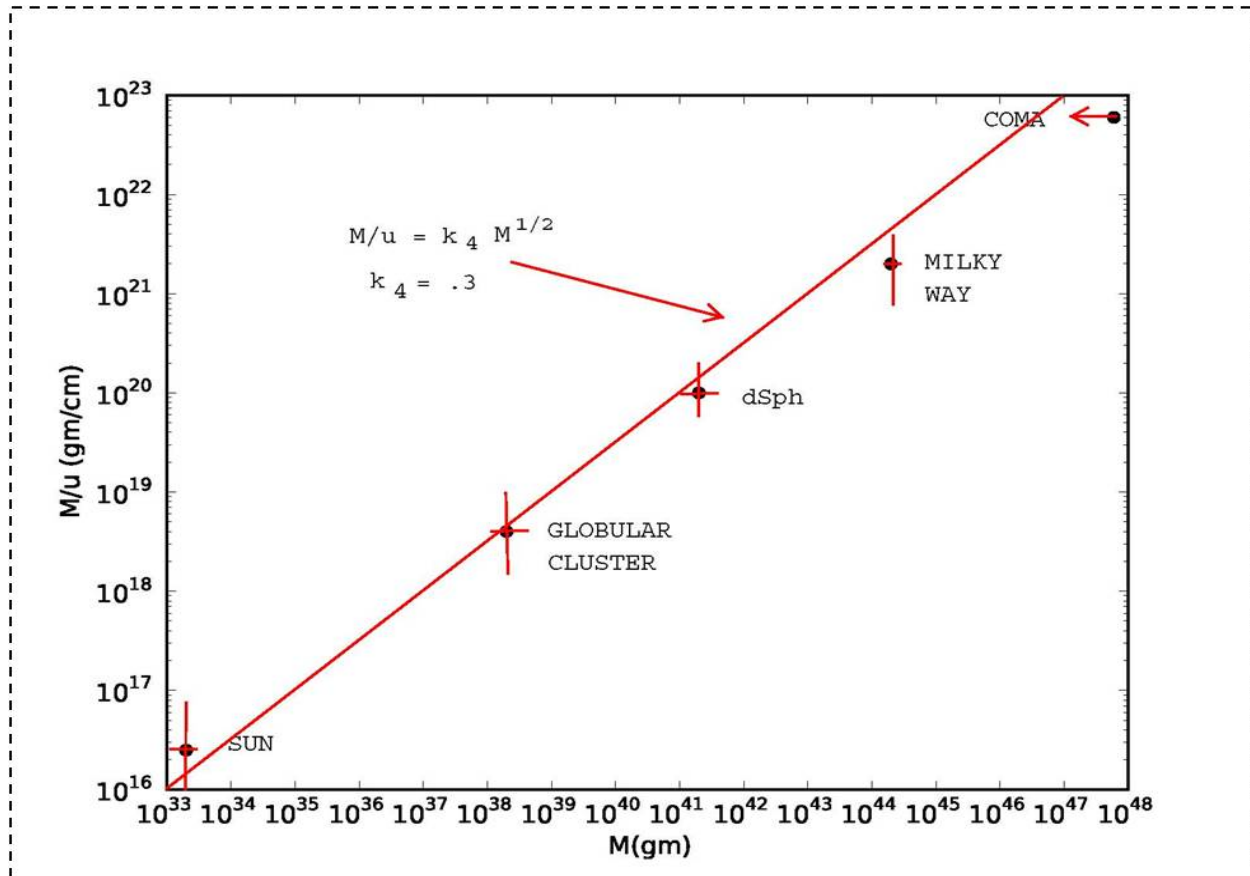


Figure 3: The resulting plot of values shown in Table 1. Errors shown are purely formal, and merely represent order of magnitude uncertainties in the quoted values.

Quite astonishingly, we see that " u " in Eq. 4 can now be transformed away, since $M/u \propto \sqrt{M}$. Thus, $\sqrt{M}/u$ becomes a universal invariant independent of mass. Even more remarkable is the quantitative value that this transformation yields:

$$|a_m| = \frac{G}{R} \cdot \frac{M_i}{u_i} = \frac{G}{R} \cdot k_4 \sqrt{M_i} = \sqrt{G \cdot k_4^2 \cdot \frac{GM_i}{R^2}} = \sqrt{a_0 \cdot a_N} \quad 5)$$

where R now replaces r_{ik} as the distance between the test particle and the most dominant source of gravity locally.

This is the exact functional form of MOND (Milgrom, 1983a),

where a_N is the Newtonian acceleration, and $a_0 = G \cdot k_4^2 \approx 6 \cdot 10^{-9} \text{ cm/s}^2$,

quite close to the value chosen purely phenomenologically by Milgrom (1983b).

Note that at $R = u_i$,

$$|a_m| = G M_i / u_i^2 = G k_4^2 = a_0, \text{ independent of } M. \quad 5a)$$

Thus, using an explicitly Machian analysis of large scale structure throughout the Universe, we can for the first time place MOND on a firm *dynamically empirical* basis. Therefore, dark matter may be unnecessary to explain (for example) the ubiquitous, flat galactic rotation curves that have puzzled astrophysicists for over half a century.

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