Ph 444   Numerical Assignment 2

Due: Tuesday, December 9, 2014

The small anisotropies in the CMB are a rich source of information on the properties of the universe. This assignment explores how this information is extracted from the observed map of CMB intensity as a function of position on the sky and then interpreted in terms of values for parameters of a model for the universe.

Files available on the class website give the antenna temperatures for either real WMAP data or for similar artificial data. The unit of the antenna temperatures in all of the files is *milliKelvins*. Each file contains data for a complete ring around the sky along a line of constant galactic latitude and the spacing between the points is about 0.139° on the sky. The files have 2048 points and each line is a separate point. The first number on the line is the angular displacement of the point around the ring in degrees (starting at 0), the second is the antenna temperature (with the mean and, possibly, other quantities subtracted — see below), and, the third, when present, is the estimated uncertainty in the antenna temperature due to noise in the measurements.

The data measured by WMAP are at a frequency of 60 GHz and along a ring in the sky with a galactic latitude of about 38°. These data are the average results for the first five years of WMAP observations. Actually, what is listed in the file is a modified antenna temperature. First, the mean temperature, which we will assume to be 2725 mK, has been subtracted. Second, both the dipole variation and a model for the contamination by galactic foreground emission have been subtracted from these data (see http://lambda.gsfc.nasa.gov/product/foreground/ for more details), leaving just the anisotropies imposed (principally) at the epoch of decoupling. Three files of WMAP data are present on the website and contain the ring at galactic latitude 38° and the two rings offset by about ±0.6° in latitude. The offsets are about twice the WMAP beam width at 60 GHz, so the three sets of data are independent.

Another file on the website gives a set of artificial antenna temperatures with a simple sinusoidal variation with location on the sky. The number of points and format of the file is the same as for the WMAP data, except that no uncertainties are present. Again, the numbers in the file are antenna temperature minus 2725 mK.

Finally, two files contain artificial data resembling the central WMAP ring but containing artificial data that consists of Gaussian random numbers with a root-mean-square (rms) dispersion equal to the measurement uncertainties of the WMAP data. The second file is the same as the first, but has been smoothed with the WMAP beam at 60 GHz. In particular each pixel was combined with its six nearest neighbors using the weights $(0.0091, 0.0500, 0.2340, 0.4138, 0.2340, 0.0500, 0.0091)$.

1. Use a spreadsheet, a computer program, or a plotting utility to plot the three sets of WMAP data as a function of position along the ring. Connect the points with

lines. The strings of data extend through about 284° because they are along a small circle at a latitude of 38° (360° × cos(38°) = 283.7°). Make the three plots for both the full angular range of the data sets and for the first 25°. It is easier to see the individual measurements in the latter plot. Can you *visually* detect any correlations between the three rings of data using your plots? In other words, do the temperatures vary in a similar way along the rings?

2. Make similar pairs of plots (full range and first 25°) for the artificial data with Gaussian random numbers both with and without smoothing. Finally, make a similar pair of plots for the sinusoidal artificial data.

3. Calculate the mean antenna temperature, $\langle T \rangle$, for each of the three sets of WMAP data; the root-mean-square (rms) variation about the mean, $\langle (T - \langle T \rangle)^2 \rangle^{1/2}$, and the rms *fractional* temperature deviation, $\langle ((T - \langle T \rangle)/T)^2 \rangle^{1/2}$. For the first two averages, use the mean-subtracted antenna temperatures (so you are actually calculating the mean and *rms* δT). For the third average add the mean of 2725 mK to the antenna temperature in the denominator of $(T - \langle T \rangle)/T$. Why are your three values of $\langle T \rangle$ not equal to zero? How do the three values compare to each other? How do the three rms variations compare with the typical measurement uncertainty per point given in the three data files?

4. Calculate the same mean, rms variation, and fractional rms variation for the two sets of artificial data composed of Gaussian noise. How closely do the mean antenna temperatures differ from zero? How do the rms variations compare to the typical measurement uncertainty per point? Calculate the same mean, rms variation, and fractional rms variation for the sinusoidal artificial data.

5. Most of the information contained in the CMB anisotropies comes from the average amplitude of the fluctuations as a function of angular scale. This information is contained in the angular correlation function defined by Ryden equation (9.46):

$$C(\theta) = \left\langle \frac{\delta T}{T}(\hat{n}) \frac{\delta T}{T}(\hat{n}') \right\rangle_{\hat{n}\cdot\hat{n}'=\cos(\theta)} . \tag{1}$$

Here the average is over all of the pairs of points on the sky with angular separations of θ. Use the sinusoidal artificial data to calculate $C(\theta)$ using all of the pairs of points in a ring with locations $(i, i+2)$, which are separated by $\theta \approx 0.28°$. Repeat for the pairs $(i, i+4)$, separated by about 0.55°, and for pairs separated by 6, 8, 10, 12, 14, 16, and 20 — this last is a separation of 2.77°. Plot your $C(\theta)$ vs. θ. You should confirm that $C(\theta)$ varies sinusoidally with the same period as the artificial data. If not, check your method/program until you find the error. Explain the behavior of $C(\theta)$ using the sinusoidal variation of the antenna temperature. A perfect correlation between pairs of points would result in a value for $C(\theta)$ equal to $\langle ((T - \langle T \rangle)/T)^2 \rangle$ (the square of the value you calculated in part 4). A perfect anti-correlation would be the negative of this value. How strong are the strongest correlations and anti-correlations seen in your $C(\theta)$ in units of the maximum possible values?

6. Use the Gaussian-random artificial data to calculate $C(\theta)$ using the pairs of points with positions $(i, i+1)$ and for pairs separated by 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 30, and 40. This last is a separation of about 5.5°. Plot this $C(\theta)$ vs. θ. Discuss the difference between this $C(\theta)$ and that for the sinusoidal data.

7. Repeat the calculation and plot of $C(\theta)$ in part 6 for the smoothed Gaussian data. Where does the maximum $C(\theta)$ occur and what is it in units of the maximum possible correlation? Explain what causes the difference between this $C(\theta)$ and that from part 6.

8. Calculate $C(\theta)$ separately for each of the three sets of WMAP data using the same angular separations as parts 6 and 7. Plot your your three $C(\theta)$s vs. θ on a single plot. How well do your three estimates of $C(\theta)$ agree? The differences are an estimate of the statistical uncertainty in your estimate of $C(\theta)$. Briefly discuss where the statistical uncertainty in your $C(\theta)$ comes from.

9. Average your three WMAP $C(\theta)$s and plot the average. The separations between the points are actually slightly different for the three sets of data, but ignore this and just average each group of three points with nearly the same θ. Where does the maximum $C(\theta)$ occur and what is it in units of the maximum possible correlation? How does your plot compare to the plot of $C(\theta)$ handed out in class, which was calculated for the entire WMAP dataset? How does your average WMAP $C(\theta)$ compare to the $C(\theta)$s for the unsmoothed and smoothed Gaussian artificial data from parts 6 and 7? Explain the origin of the differences between the average WMAP $C(\theta)$ and those of the two Gaussian artifical datasets in as much detail as you can. In particular, discuss the evidence that the increasing values of the WMAP $C(\theta)$ for $\theta < 1°$ are NOT explainable solely by the angular resolution of the WMAP antennas.

10. The shape of the WMAP $C(\theta)$ reflects the angular size of the temperature fluctuations in the CMB caused by primordial energy density fluctuations at the time of decoupling. In particular, the shape of $C(\theta)$ is determined by the angular size of the particle horizon at the time of decoupling, which corresponds to the first peak in the angular power spectrum of $C(\theta)$ (*e.g.*, Figure 9.6 in Ryden and the similar plot handed out in class). Calculating the full power spectrum is beyond the scope of a homework set. However, because the temperature fluctuations on the angular scale of the first peak are dominant, we can approximate $C(\theta)$ for $\theta \leq 1°$ with the function

$$A\cos(\theta(360°/W)). \tag{2}$$

Here A is an amplitude that depends on the strength of the fluctuations in the antenna temperature and $W/2$ is the size (the full-width at half-maximum) of the fluctuations (*i.e.*, compare to the sinusoidal temperature fluctuations in part 5). We can think of this as the dominant term in the spherical harmonic representation of $C(\theta)$. Try adjusting A and W in equation 2 to produce the best match to the first seven points of your average WMAP $C(\theta)$ (*i.e.*, $\theta < 1°$). How does your best value for $W/2$ compare to the angular size corresponding to the first peak in the power spectrum?