Ph 444 Problem Set 5

Due: Friday, October 17, 2014

1. a) Find an explicit expression for the proper horizon distance, as a function of time, for both matter-dominated and radiation dominated flat Robertson-Walker universes.

b) Evaluate the matter dominated expression for the current time, and comment on the discrepancy between this value, and our current estimate of 14 Gpc.

2. Consider a cosmological model described by the line element

$$ds^2 = -c^2\, dt^2 + (t/t_0)\left[dx^2 + dy^2 + dz^2\right] \tag{1}$$

a) Is this model open, closed, or flat?

b) Is this a matter dominated universe? Explain.

c) Assuming the Friedmann equation holds for this universe, find $\rho\,(t)$.

3. It is often quite convenient to define a new time coordinate η such that

$$dt = a\,(t)\,d\eta \tag{2}$$

This is called *conformal time*. With this definition, since time is now scaled with the expansion of the universe, radial light rays (for example) move on 45° lines, just as in Special Relativity.

a) Show that if the parameter η is used as a time coordinate, the closed Robertson-Walker metric takes the form:

$$ds^2 = a^2\,(\eta)\left\{ - c^2 d\eta^2 + R^2\left[d\chi^2 + sin^2\chi\left(d\theta^2 + sin^2\theta d\phi^2\right)\right]\right\} \tag{3}$$

where $\chi \equiv r/R$, with R equal to the curvature.

b) Draw an $\eta - \chi$ spacetime diagram indicating the big bang, the big crunch, and the past light cone of a comoving observer at the origin at the moment of maximum expansion.

c) Is there time before the big crunch for the observer to receive information from all parts of this spatially finite universe, or are there parts of it that he or she is doomed never to see?

d) Could an observer traverse the entire circumference of the universe in the time between the big bang and the big crunch?

4. a) Solve the Friedmann equation to obtain an explicit expression for the scale factor a(t) in the case of a flat universe which contains only vacuum energy and matter (i.e. there is no radiation present). Express your answer in terms of H_0, Ω_m, *and* $\Omega_\Lambda = 1 - \Omega_m$. (*Hint:* you might find the substitution $y = a^{3/2}$ to be helpful for the integration.)

b) How large would Ω_Λ have to be for the universe to be accelerating ($\ddot{a} > 0$) at the present time?

c) Find an explicit expression for the age of the universe t_0 as a function of H_0 *and* Ω_Λ.