

Ph 444 Problem Set 4

Due: Friday, October 10, 2014

1. In the same way that we incorporated the $\Lambda/3$ term from the Friedmann equation into a term involving ϵ_Λ , we can incorporate the term involving Λ from Poisson's equation into an "effective" potential.

a) Find an additional term for the potential that will turn $\Lambda + \nabla^2\phi = 4\pi G\rho$ into $\nabla^2\phi_{eff} = 4\pi G\rho$.

b) Show that this term leads to a force F_Λ that is radially outward for $\Lambda > 0$.

c) Incorporate the result from a) into the Newtonian version of Friedmann's equation, and show that it leads to additional term involving $\Lambda/3$, exactly analagous to the result of the relativistically correct version.

2. Channel 6 on your television set (if you have one; I don't!) consists of radio waves with wavelengths $3.41\text{ m} \leq \lambda \leq 3.66\text{ m}$. Consider a 25,000-watt television station located 70 km from your home. Use Equation 2.25 from Ryden for the energy density of blackbody radiation to estimate the ratio of the number of channel 6 photons to the number of CMB photons that your television antenna picks up in this wavelength band. (*Hint:* For the TV broadcast, remember that the time-averaged Poynting vector $\langle S \rangle$ not only gives the monochromatic flux of radiation at any distance, but is related to the energy density ϵ via $\epsilon = \langle S \rangle / c$.)

3. Some quantities obey an exponential time-behavior of the form $f(t) = f_0 e^{t/\tau}$, where τ is the *characteristic time* for the system under consideration.

a) Show that

$$\tau = \left(\frac{1}{f} \frac{df}{dt} \right)^{-1}.$$

This expression can be used to define a characteristic time for any function, regardless of whether its behavior is exponential.

b) Use the scale factor, $a(t)$, to show that the characteristic time for the expansion of the universe is $\tau_{exp}(t) = 1/H(t)$.

c) Assuming a flat universe containing only matter and radiation, find an expression (valid in both the radiation era and the matter era) for the characteristic expansion time τ_{exp} as a function of the scale factor $a(t)$.

4. In class, we derived a set of two equations, specifically for non-relativistic matter, which we used to show that the early universe becomes flat in the limit of $z \rightarrow \infty$. But the universe is radiation dominated in early times! So, use a similar procedure

to demonstrate that the universe also becomes flat if it is dominated by relativistic particles.