Ph 444 Numerical Assignment 1

Due: Friday, November 7, 2014

The fluctuations in the cosmic microwave background (CMB) radiation provide some of our most powerful constraints on the properties of the universe. We will see later in the course that an important length scale which is imprinted on the fluctuations is the particle horizon at the time when the matter and CMB radiation last interacted. This epoch of “last scattering” occurred at a redshift of about $z = 1090$. The particle horizon is the scale on which causal effects — sound waves in the case of the CMB — can operate. Your goal in this assignment is to see how the observable angular diameter of the particle horizon at the time of last scattering depends on some of the properties of the universe. We will assume that the redshift of last scattering is fixed. This is an approximation, but actually a very good one for reasons that we will study later. Assume that the universe today contains radiation with $\Omega_{r,0}$, matter with $\Omega_{m,0}$, and a cosmological constant with $\Omega_{\Lambda,0}$ and that $\Omega_0 = \Omega_{r,0} + \Omega_{m,0} + \Omega_{\Lambda,0}$ does not necessarily equal one.

1. Write a computer program or spreadsheet that calculates the particle horizon, $d_{hor}(z)$, in units of c/H_0, at the epoch corresponding to a redshift of $z = 1090$. Show the expression that you are using for d_{hor} and describe how you are integrating it. What is the size of d_{hor} for the current best estimates of $\Omega_{r,0} = 8.4\times10^{-5}$, $\Omega_{m,0} = 0.27$, $\Omega_{\Lambda,0} = 0.73$, and $H_0 = 71$ km/s/Mpc? Express your answer in Mpc at the epoch of $z = 1090$. What distance today corresponds to the same comoving distance as $d_{hor}(z = 1090)$?

2. What is the fractional change of d_{hor} for $\pm10\%$ changes in each of $\Omega_{r,0}$, $\Omega_{m,0}$, and $\Omega_{\Lambda,0}$ about the standard model given in part 1? Explain in words what causes the size of the change in d_{hor} to be different for a 10% change in the three parameters.

3. What is the fractional change of d_{hor} if $\Omega_{\Lambda,0}$ is changed by $\pm10\%$ and $\Omega_{m,0}$ is also changed so as to keep $\Omega_0 = 1.0$?

4. Write a computer program or spreadsheet that calculates the angular-diameter distance, $d_a(z)$, to an object with a redshift of $z = 1090$ in units of c/H_0. Show the expression that you are using for d_a and describe how you are integrating it. What is the size of d_a in Mpc for the parameters of part 1 and what is the resulting angular size, θ_{hor} (in degrees), of the d_{hor} from part 1?

5. What is the fractional change of d_a for $\pm10\%$ changes in each of $\Omega_{r,0}$, $\Omega_{m,0}$, and $\Omega_{\Lambda,0}$ about the standard model given in part 1? Explain in words what causes the size of the change in d_a to be different for a 10% change in the three parameters.

6. Calculate the fractional change in θ_{hor}, the observed angular size of the horizon at $z = 1090$, for a $\pm10\%$ change in each of the Ω’s using your results from parts 2 and 5.

7. What is the fractional change of d_a if $\Omega_{\Lambda,0}$ is changed by $\pm 10\%$ and $\Omega_{m,0}$ is also changed so as to keep $\Omega_0 = 1.0$? Calculate the corresponding fractional change in θ_{hor}.

8. Modify your program/method for calculating the angular-diameter distance, $d_a(z)$, for the case where the cosmological constant is replaced by quintessence with $w = -0.5$. Show the expression that you are using for d_a. Assume the same standard cosmological parameters as before, except that now $\Omega_{q,0} = 0.73$. What is the fractional change of $d_a(1090)$ from the $w = -1$ case? Explain in words why the horizon distance at z=1090 will be unchanged by changing w from -1 to -0.5. Calculate the fractional change in θ_{hor} caused by a change from a cosmological constant to quintessence.